Faculty of Engineering & the Built Environment

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An investigation into the effects of early propping removal on the deflection of reinforced concrete beams

Dissertation presented in partial fulfilment of the requirements for the degree of Master of Engineering in Structural Engineering

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Abstract

In today's fast paced construction industry, there is an ever present need to increase productivity and to complete projects as quickly as possible. Reinforced concrete is a popular and widely used construction material. However it has the unfortunate drawback in that the concrete requires time to set and gain sufficient strength before loads may be applied and the formwork and props can be removed. It is therefore desirable to keep propping times to a minimum. If the propping is removed too early, there is a risk of the member deflecting excessively and exceeding the maximum allowable limits, or in severe cases it could even lead to a structural failure or collapse.

The SANS 2001 code provides recommended propping times for beams and slabs, which can be used as a guideline by building contractors and structural designers. These propping times present a universal approach, which does not consider all the factors that affect deflection. This simplified approach may be considered to be conservative as shorter propping durations could be possible without a loss in performance.

The aim of this dissertation is to look into the effects of early propping removal on the long-term deflections of concrete members. This was done by modelling the deflection of a typical reinforced concrete beam at different ages of loading, using three code-based deflection calculation methods. The codes that were used are the South African National Standard (SANS), Eurocode (EC2) and American Concrete Institute code (ACI 318). A detailed literature-based investigation was conducted to determine the factors which affect deflection in reinforced concrete members, as well as the theory behind the code-based deflection calculation procedures. This was followed by the modelling of deflections using the above-mentioned methods. Three case studies were performed to determine the effects of early propping removal under different scenarios. The first case study only deals with the effects of early age loading on long-term deflection. Early age in this case was considered to be any duration that is less than the recommended propping time given in SANS 2001. As an added point of interest, two different concrete mixes were used, made with two different types of cement. The second case study compares the effect that different levels of relative humidity have on the long term deflection at early ages of loading. Lastly, the effects of concrete strength on long-term deflections at early ages of loading was modelled.

The results of the first case study indicated that for a certain application, a reduction in propping time may be possible without exceeding the maximum allowable deflection limits specified by

the SANS 10160 code. In the second and third case study is was observed that both the relative humidity and concrete strength respectively have an effect on the long term deflection and therefore also influence the propping time. Especially the relative humidity seems to have a significant effect on the change in deflection. The concrete strength on the other hand only seemed to have a more moderate effect on the long-term deflection.

A simple propping time calculation tool which is based on the deflection modelling methods used for the three case studies was introduced. This tool allows structural designers to easily calculate the estimated propping time for various flexural members. It allows the designer to model the propping time for different member sizes, support configurations, material properties, environmental conditions and load cases.

The study concluded that based on the obtained estimated deflection values using the code-based methods, the propping times provided in the SANS 2001 code may in certain applications be too conservative. According to the results obtained, it may be possible to reduce the propping duration under certain conditions and for certain applications. It was suggested that an alternative method should be developed which would allow structural designers to determine the required propping time more accurately.

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Notation

Δ	-	Short-term (initial) deflection (Eurocode)
Δ_{∞}	-	Long-term creep deflection (SANS)
Δ_{cs}	-	Deflection due to shrinkage
Δ_{i}	-	Short-term (initial) deflection (ACI, SANS)
Δ_{l}	-	Long-term deflection (ACI)
Δt_{i}	-	Number of days when temperature T prevails
$\frac{1}{\mathrm{r}}$	-	Curvature of a flexural member at mid-span
$\frac{1}{r_{cs}}$	-	Curvature due to shrinkage for a flexural member (Eurocode)
$\frac{1}{r_{n}}$	-	Initial or long-term flexural curvature (Eurocode)
$\frac{1}{r_{t,QP}}$	-	Total curvature due to shrinkage and creep (Eurocode)
a	-	Constant used to describe the strength gain development of the concrete (ACI)
A	-	Cross sectional area of flexural member (SANS)
A_{c}	-	Cross sectional area of flexural member (Eurocode)
A_{s}	-	Cross sectional area of tension reinforcing steel
A'_{s}	-	Cross sectional area of compression reinforcing steel
A_{s2}	-	Cross sectional area of compression reinforcing steel (Eurocode)
b	- -	Width of a section (ACI, Eurocode & SANS) Constant used to describe the strength gain development of the concrete (ACI)
B	-	
c	-	Cement content of concrete kg/m³ (ACI)
d	-	Effective depth to centre of tension reinforcement
d_{2}	-	Effective depth to centre of compression reinforcement (Eurocode)
d'	-	Effective depth to centre of compression reinforcement (ACI & SANS)
E_{c}	-	Modulus of elasticity of concrete
$E_{c,28}$	-	Tangent modulus of elasticity of normal weight concrete at 28 days (Eurocode)
$E_{c.eff}$	-	Effective modulus of elasticity of concrete (Eurocode)
E_{cm}	-	Secant modulus of elasticity of concrete (Eurocode)
$E_{cm(t)}$	-	Secant modulus of elasticity of concrete at time t (Eurocode)
$E_{c,t}$	-	Tangent modulus of elasticity of normal weight concrete at time t (Eurocode)
E_{eff}	-	Effective modulus of elasticity of concrete (Eurocode)
E_{s}	-	Design value of modulus of elasticity of reinforcing steel
f	-	Constant for a given member shape and size (ACI)
f_{c}'	-	Specified compressive cylinder strength of concrete at 28 days
f_{ck}	-	Characteristic compressive cylinder strength of concrete at 28 days
f_{cm}	-	Mean compressive cylinder strength of concrete (Eurocode)
f_{cm28}	-	Concrete mean compressive cylinder strength at 28 days (ACI)
f_{cmo}	-	Constant given as 10 MPa (Eurocode)
$f_{cm(t)}$	-	Mean compressive cylinder strength of concrete at age t (Eurocode)
f_{cmt}	-	Concrete mean compressive cylinder strength at age t (ACI)
f_{ctm}	-	Mean value of axial tensile strength of concrete (Eurocode)
$f_{ctm(t)}$	-	Mean value of axial tensile strength of concrete at age t (Eurocode)

f_{cu}	-	Compressive cube strength of concrete (SANS)
$f_{cu,28}$	-	Compressive cube strength of concrete at 28 days (SANS)
$f_{cu,t}$	-	Compressive cube strength of concrete at age t (SANS)
f_r	-	Modulus of rupture of concrete (ACI &SANS)
h	- -	Overall depth of a cross-section (ACI, Eurocode & SANS) Relative humidity expressed as a decimal (ACI)
h_0	-	Notional member size (Eurocode)
I_c	-	Moment of inertia of a cracked (equivalent) concrete section (Eurocode)
I_{cr}	-	Moment of inertia of a cracked (equivalent) concrete section (ACI & SANS)
I_e	-	Effective moment of inertia of concrete section
I_g	-	Moment of inertia of concrete section ignoring reinforcement (SANS)
I_u	-	Moment of inertia of an uncracked concrete section
K	-	Deflection coefficient dependent on the bending moment diagram of the flexural member
K_0	-	Aggregate stiffness factor related to the elastic modulus of the aggregate and its volume concentration
k_{cs}	-	Factor used to determine the shrinkage deflection (SANS)
k_h	-	Coefficient depending on the notional size
k_r	-	Ratio of the initial neutral axis depth to the neutral axis depth due to creep
K_{sh}	-	Shrinkage deflection coefficient dependent on the shrinkage curvature shape
l	-	Span of a member between supports (SANS)
L	-	Span of a member between supports (ACI & Eurocode)
M	-	Bending moment
M_a	-	Maximum bending moment in the element corresponding with the deflection situation under consideration
M_{cr}	-	Cracking moment
M_{QP}	-	Moment due to quasi-permanent action
M_s	-	Maximum applied bending moment due to service loads
r	-	
RH	-	Relative humidity of the ambient environment in %
RH_0	-	Constant given as 100% (Eurocode)
s	- -	Slump measured in mm (ACI) Coefficient dependent on the cement type (Eurocode)
S	-	Surface area used in volume to surface area calculation
S_c	-	First moment of area of the reinforcement about the centroid of the cracked section
S_u	-	First moment of the area of reinforcement about the centroid of the uncracked section
t	-	Age of concrete in days at time being considered
t_0	-	Age of concrete at time of loading in days
$t_{0,T}$	-	Temperature adjusted age of concrete at loading in days
t_c	-	Age of concrete in days, when drying starts after moist curing
t_s	-	Age of concrete in days at the start of drying shrinkage
$T(\Delta t_i)$	-	Temperature in °C during time period Δt_i
u	-	Perimeter of concrete cross section
V	-	Volume of section used in volume to surface area calculation
w_c	-	Specified density of concrete
x	-	Depth from compression face to neutral axis
x_c	-	Distance of the neutral axis of the cracked section to the compression face

x_g	-	Centreline through the height of a section
x_i	-	Ratio of neutral axis depth to effective depth of the cracked element
x_u	-	Distance of the neutral axis of the uncracked section to the compression face (Eurocode)
y_t	-	Distance from the centroidal axis of the concrete section to the tension face
α	-	Coefficient; air content in percent (ACI)
$\alpha_1, \alpha_2, \alpha_3$	-	Coefficients to consider the effects of concrete strength on the development of creep with time
α_{ds1}	-	Coefficient dependent on the cement type
α_{ds2}	-	Coefficient dependent on the cement type
α_e	-	Modular ratio expressed as the ratio between the elastic modulus of steel to the elastic modulus of concrete
$\beta(f_{cm})$	-	Factor which allows for the effects of concrete strength on the notional creep coefficient
$\beta(t_0)$	-	Factor which allows for the effects of concrete age at loading on the notional creep coefficient
$\beta_{as}(t)$	-	Coefficient related to the effects of age of concrete (in days) on autogenous shrinkage
$\beta_c(t, t_0)$	-	Coefficient to describe the development of creep with time after loading
$\beta_{cc(t)}$	-	Coefficient which depends on the age of concrete t
$\beta_{ds}(t, t_s)$	-	Coefficient related to the effects of age of concrete on the drying shrinkage
β_H	-	Coefficient depending on the relative humidity and the notional member size
β_{RH}	-	Coefficient related to the effects of relative humidity to the basic drying shrinkage strain
γ_c	-	Cumulative product of correction factors for creep coefficient calculation
$\gamma_{c,RH}$	-	Ambient relative humidity factor
$\gamma_{c,s}$	-	Slump factor
$\gamma_{c,t0}$	-	Age of loading factor
$\gamma_{c,vs}$	-	Coefficient which allows for the member size in terms of volume-surface area
$\gamma_{c,\alpha}$	-	Air content factor
$\gamma_{c,\Psi}$	-	Fine aggregate factor
γ_{sh}	-	Cumulative product of correction factors for shrinkage strain calculation
$\gamma_{sh,c}$	-	Cement content factor
$\gamma_{sh,RH}$	-	Ambient relative humidity coefficient
$\gamma_{sh,s}$	-	Slump factor
$\gamma_{sh,tc}$	-	Coefficient for curing times
$\gamma_{sh,vs}$	-	Coefficient which allows for the member size in terms of volume-surface area
$\gamma_{sh,\alpha}$	-	Air content factor
$\gamma_{sh,\Psi}$	-	Fine aggregate factor
δ_{QP}	-	Deflection due to quasi-permanent loading
ζ	-	Distribution coefficient allowing for tension stiffening
ε_{ca}	-	Autogenous shrinkage strain
$\varepsilon_{ca}(\infty)$	-	Final autogenous shrinkage strain
$\varepsilon_{ca}(t)$	-	Time dependent autogenous shrinkage strain
ε_{cd}	-	Drying shrinkage strain
$\varepsilon_{cd,0}$	-	Basic drying shrinkage strain

$\varepsilon_{cd}(t)$	-	Time dependent drying shrinkage strain
$\varepsilon_{cs}(t, t_c)$	-	Time dependent shrinkage strain at age of concrete t in days (ACI)
ε_s	-	Free shrinkage strain of concrete (SANS)
ε_{sh}	-	Shrinkage strain (ACI)
ε_{shu}	-	Ultimate shrinkage strain (ACI)
λ	-	Deflection multiplier for long term deflection (SANS)
λ_Δ	-	Deflection multiplier for long term deflection (ACI)
ξ	-	Time dependent deflection factor for sustained load
ρ	-	Ratio of tension reinforcement provided relative to the area of the concrete
ρ'	-	Ratio of compression reinforcement provided relative to the area of the concrete
ϕ	-	Creep coefficient (ACI)
ϕ_u	-	Ultimate creep coefficient (ACI)
$\varphi(\infty, t_0)$	-	Final value of creep coefficient
φ_{RH}	-	Factor to allow for the effects of relative humidity on the notional creep coefficient
φ_0	-	Notional creep coefficient (Eurocode)
Φ	-	Creep strain divided by the initial strain; Creep factor considering age of concrete at loading, humidity, surface-to-volume ratio, etc.
Ψ	-	Ratio of fine aggregate to total aggregate by weight expressed as a percentage

1. Introduction

1.1 Background

In today's fast-paced construction industry, there is an ongoing need to increase production outputs by streamlining processes wherever possible. Reinforced concrete (RC) is a popular and widely used building material, but it has the drawback that it requires sufficient time to develop the required strength to resist the intended load. Removal of propping and loading of RC beams and slabs can only take place when adequate strength and maturity has been obtained. If props are removed too early, it could lead to excessive deflections. However, it is generally desirable to keep the propping time to a minimum in order to save time and minimise costs during the construction phase. Table 1 below provides recommended propping times for various concrete member types according to the South African National Standards (SANS). This table is to be used in the absence of any qualitative data, such as the early strength development of the concrete being used, which would otherwise allow the structural designer to determine the suitable propping duration.

Table 1: Recommended propping times according to SANS 2001 [1]

1	2	3	4	5	6	7	8	9	10
Formwork to structural member	**Strength class of cemet**								
	42,5 R or higher			**CEM I and CEM II A-S, D, P, Q, V,A, W, T, L, LL, M and blends of CEM I with 20% or less ground granulated blast-furnice slag or fly ash**			**CEM II B-S,P, Q, V,W, T, L,LL, M; CEM III, CEMIV and CEMV and blends of CEM I with more than 20% ground granulated blast-furnace slag or fly ash**		
	Minimum time before removal of formwork d								
	Weather								
	Hot or normal	**Cool**	**Cold**	**Hot or normal**	**Cool**	**Cold**	**Hot or normal**	**Cool**	**Cold**
Beam sides, walls and unloaded	0.5	0.75	1	0.75	1.25	1.5	2	3	4
Slabs with props left underneath	2	3	4	4	5.5	7	6	8	10
Beam soffit with props left underneath and ribs with a ribbed floor construction	3	4	5	7	9.5	12	10	13.5	17
Slab props including cantilevers	5	7	9	10	13.5	17	10	13.5	17
Beam props including cantilevers	7	9.5	12	14	17.5	21	14	17.5	21
NOTE In cool weather stripping times may be determined by interpolation between the periods specified for normal and cold weather.									
[a] A day is taken as 24h.									

The first column of the table provides a description of the formwork components for various RC elements. The minimum duration before formwork and props can be removed is measured in days and has been grouped according to the class of cement used. The cement types are arranged according to strength class and in order of fastest to slowest in terms of strength development under normal conditions. For each cement type, the duration is further divided according to the prevailing weather conditions during that time. These are separated into three categories: Hot or normal; Cool; Cold. These categories allow for the effects of temperature on the hardening of concrete. The code provides definitions for each temperature category, which have been summarised in Table 2.

Table 2: Temperature ranges for weather conditions as per SANS 2001

Weather	Temperature Range
Hot	Temp > 32°C
Normal	15°C < Temp < 32°C
Cool	5°C < Temp < 15°C
Cold	Temp ≤ 5°C

The code also states that the propping durations may be reduced if the early concrete strength is assessed, based on cube tests which have been cured for the same duration and at the same temperature as the concrete in the element. This suggests that the propping time given by the SANS code is linked to the compressive strength of the concrete and that the props may be removed as soon as the concrete has gained its specified design strength.

International codes provide different means of determining the propping duration. The design code developed by the American Concrete Institute (ACI), for example, suggests that the propping may only be removed if it can be proven by means of structural analysis that the member will be strong enough to support itself and any load that it might have to resist, without exceeding the maximum recommended deflection limits [2]. Data used in the analysis should be obtained from compression tests performed on field-cured cylinders. Other approved procedures, such as penetration resistance, pullout strength and maturity index measurements may also be used, provided that the data obtained from these procedures correlate with the compressive strengths of the cylinders or drilled core samples. This indicates once again that the propping time is linked to the concrete compressive strength. The ACI also provides some propping time guidelines in its formwork design guide, ACI 347. These have been summarised in Table 3 below [3]. The values in the table may be used when no specification for the minimum strength of concrete at the time of stripping is provided. However, the ACI 347 document also states that these propping durations may be adjusted at the discretion of the engineer, depending on the type of cement being used, ambient temperature and the use of retarding admixtures.

Table 3: Propping duration guidelines according to ACI 347

Description of structural member type	Structural live load less than structural dead load	Structural live load more than structural dead load
Arch centres	14 days	7 days
Joist, beam or girder soffits:		
Under 3 m clear span between structural supports	7 days*	4 days
3 to 6 m clear span between structural supports	14 days*	7 days
Over 6 m clear span between structural supports	21 days*	14 days
One-way floor slabs:		
Under 3 m clear span between structural supports	4 days*	3 days
3 to 6 m clear span between structural supports	7 days*	4 days
Over 6 m clear span between structural supports	10 days*	7 days
* Where forms can be removed without disturbing shores, use half of value shown but not less than 3 days		

1.2 Project Motivations

Early removal of formwork and propping of RC members can significantly reduce construction time and associated cost. However, premature removal of props can also cause excessive long-term deformations which might exceed the serviceability design limits. Deflection of RC members is affected by a multitude of factors, particularly the long-term effects, for which creep and shrinkage play an important role. These in turn are affected by various intrinsic and extrinsic factors [4, 5] including concrete ingredients and proportions, curing conditions, relative humidity and age at loading. In particular, the latter aspect relates to the propping time of the members. Accurate modelling of beam deflections during the early-age strength development of concrete can be used to determine if a reduction of the propping duration is possible, and if so, at which point in the process it should be undertaken.

1.3 Research Objectives, Methodology and Limitations

The aim of this investigation is to determine how the short- and/or long-term deflection of RC beams will be affected if propping is removed at an earlier age than recommended in the propping time table of the SANS 2001 design code.

The deflection of RC members is affected by many factors. To a large extent this is due to the complex nature of concrete, which makes it difficult to determine its exact material properties. The factors which affect the material properties of concrete will first need to be discussed in detail to highlight their possible effect on deflection. All the information for this discussion will be drawn from existing literature.

Various design codes have recommended calculation procedures, which are supposed to assist the structural designer to estimate the deflection of RC beams and slabs. Three of these codes will be used to model deflection at various ages of loading. The codes used in the comparison are SANS 10100, ACI 318 and Eurocode EC2. Each code provides a different method for calculating the short and long term deflection. In the second part of this research, the methods prescribed by the three codes will be examined in detail to determine how they have been derived and how they incorporate the various factors which affect deflection. The accuracy of these methods will also be discussed. This will be based on existing research that has been done in this field.

The code-based deflection modelling will be applied to a typical continuous beam type. Other beam types or slabs will not be considered. The aim of the modelling is to determine how the individual deflection calculation procedures assess the material properties, as well as the creep and shrinkage effects and how these are then incorporated into the deflection calculation process. Both the immediate and long term deflection will be modelled. The aim is to determine how the estimated deflection values obtained from each code are affected when the age at first loading (i.e. the time to propping removal) is reduced. In addition to this, it will be determined what effects the relative humidity and concrete strength have on the deflection at different ages. The code-based models are used as a tool to demonstrate this, since they are readily available to designers and their intended use is to allow designers to roughly estimate the deflection. The obtained results will be compared to the recommended propping time table in SANS 2001. One of the practical outcomes of this research will be to develop a propping time calculation tool, which is able to calculate the estimated deflection of RC beams according to each of the code-based methods investigated in this dissertation.

2. Literature review – Modelling of Deflection of Concrete Members

The construction of beams and slabs in multi-storey reinforced concrete structures revolves around a well-co-ordinated process which involves the erection of formwork and staging, installation of reinforcement, casting of concrete and subsequently removal of the formwork and props. This latter aspect may take place in two separate operations. The formwork may already be removed as soon as the concrete has gained sufficient strength so that no plucking of the surface occurs during the formwork removal process.[6, 7] The timing for the removal of the props on the other hand needs to be considered more carefully. As soon as the props are removed, the structural member will undergo deflection, as it starts to support its own weight, as well as any additional permanent and imposed loads that are acting on it. The removal of props may only take place once the concrete has gained sufficient strength to be able to support its own weight, as well as the loads acting on it, without deflecting excessively. Even though a structural member may be adequate to support the applied load, it may still deflect excessively. Careful consideration therefore needs to be given to the magnitude of the applied loads and deflection, when determining the appropriate timing for the removal of props.[7]

This chapter discusses how deflection develops in flexural RC members, the theory behind common deflection calculation methods given in SANS10100, ACI318 and Eurocode 2, and the factors which affect the deflection. This will be followed by an investigation into how these design codes have developed their deflection estimation procedures, and what inferences can be made from the findings.

2.1 Deflection Limits in Structures

Deflections need to be kept within acceptable limits in order to avoid adversely affecting the appearance and efficiency of the structure [8]. Excessive deflections can lead to excessive cracking in flexural concrete members, which may affect the durability of the structure. The serviceability limit states (SLS) set out in various building design codes specify recommended maximum allowable deflections and crack widths to adhere to [2, 9].

In order to ensure that the serviceability limits are met, design codes provide prescriptive procedures for controlling the deflections and crack widths. For the purpose of this study, crack width limits will not be discussed, as the focus is solely on the deflection of flexural concrete members. One way in which codes allow structural designers to determine if the deflection limits are met is by restricting the span/effective depth ratios. In cases where the span/depth ratio limits are exceeded, codes provide calculation procedures which may be used by the structural designer to calculate the expected deflection of beams and slabs. If the calculated deflection value does not exceed the recommended maximum value, then the serviceability limits for deflection are deemed to have been met.

The deflection calculation procedures for three such codes, ACI, Eurocode and SANS are discussed in more detail in Section 3. Estimating the deflection of RC members accurately can be challenging due to the nature of the material and the various factors which can alter the deflection process. These challenges are discussed in the following sections.

2.2 Deflections in Reinforced Concrete Members

In order to fully understand the complexity involved in trying to model the deflection of concrete beams and slabs, one must first understand the processes that take place within a member during deflection.

When the temporary supports of a newly cast flexural RC member are removed, initial deflection occurs as the member adjusts to support its own weight for the first time. At low load levels, the member will undergo elastic deformation. As the load increases, flexural cracking may begin to occur. This will cause an immediate reduction in the member stiffness. Over an extended period of time and under relatively constant load, initial deflection will increase due to shrinkage and creep effects and further cracking. The cracking is a result of tensile stress induced by flexure, shrinkage and thermal effects exceeding the tensile strength of the concrete. If a high level of concrete tensile stress is maintained, more and more cracks will form, resulting in increased long-term deflection [10].

Initial or short-term deflection is affected by the modulus of elasticity of the concrete (E_c), load distribution and support conditions, cross-sectional properties, load levels and degree of cracking along the length of the member. [11]

Long-term deflection occurs over extended periods of time and is primarily affected by shrinkage and creep effects. The factors affecting creep and shrinkage will be explained in Section 2.3. The total long-term deflection can be up to two or three times greater than short-term deflection. [12] Deflection in flexural members, such as beams or slabs can gradually increase over a period of many years. [13]

2.2.1 Beam Moment-curvature Relationship and Elastic Theory

The magnitude of the deflection of RC beams is influenced by various factors, including the geometry of the member (i.e. cross-section, span), support conditions, the load applied to it, as well as the material properties of the member. The beam moment-curvature relationship has been developed for flexural members, to link the applied moments (M), elastic modulus (E) and second moment of area (I) to curvature $\left(\frac{1}{r}\right)$ (where r is the radius of the deflected shape). In an elastic system the curvature is equal to the $\frac{M}{EI}$ diagram.[8] Based on this, the following curvature expression has been developed. [8]

$$\frac{1}{r} = \frac{M}{EI} \tag{2.1}$$

The deflection can be calculated from Equation 2.1 through the use of a numerical integration technique. An alternative, simplified approach would be to use the following equation.[14]

$$\Delta = KL^2 \frac{M}{E_c I_e} = \mathrm{KL}^2 \frac{1}{\mathrm{r}} \tag{2.2}$$

where K is a deflection coefficient based on the bending moment diagram for flexural members.

In order to calculate the deflection accurately, the designer needs to have a true representation of these values. In reinforced concrete structures this is especially challenging as the material is neither elastic, nor homogenous. This makes it difficult to calculate deflection to a great degree of accuracy. The SANS 10160 code provides guidelines for the allowable deflections for various scenarios and applications. Appendix A contains some examples of the deflection limits for various applications, as given in SANS 10160.

When a RC beam or slab deflects, tensile stresses are present within the tension face of the member, while the opposite face of the section will experience a corresponding compression. For sagging members, tension and compression will occur at the bottom and top faces respectively. Once the tensile stresses exceed the tensile strength of the concrete, cracks are formed which lead to an immediate reduction in stiffness. As the member deflects, additional tensile forces are taken up by the steel reinforcement. The development of cracks and reduction in stiffness as described above therefore needs to be accounted for during deflection calculations.

The elastic theory is used to empirically derive expressions for different stages of cracking. Three different cases of cracked sections have been described [8]: The cracked section (Case 1), the uncracked section (Case 2) and the partially cracked section (Case 3).

The following paragraph will describe each case in more detail as set out in [8].

Case 1: The Cracked Section

The following simplifying assumptions are made for the cross-section of a beam subjected to a bending moment M, as shown in Figure 1.

(a) Plane sections remain plane. Strains vary linearly with distances from the neutral axis.
(b) Stresses in the steel and concrete are proportional to the strains.
(c) The concrete is cracked up to the neutral axis and therefore no tensile stress exists in the concrete below it.

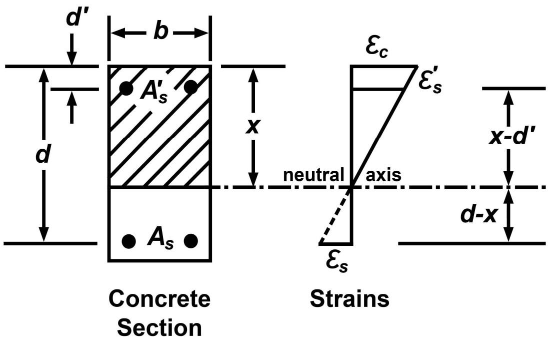

Figure 1: The cracked section for reinforced concrete [8]

Based on the first assumption, it is possible to express the steel strain in terms of the concrete strain ε_c on the compression face.

$$\varepsilon'_s = \frac{x-d'}{x}\varepsilon_c \tag{2.3}$$

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c \tag{2.4}$$

From the second assumption, the concrete stress f_c in the compression face, the tension steel stress f_s and the compression steel stress f'_c are expressed as follows:

$$f_c = E_c\varepsilon_c \tag{2.5}$$

$$f'_s = E_s\varepsilon'_s = \alpha_c E_c\varepsilon'_s \tag{2.6}$$

$$f_s = E_s\varepsilon_s = \alpha_c E_c \varepsilon_s \qquad (2.7)$$

E_s and E_c are the moduli of elasticity of the reinforcement and concrete respectively and α_c is the modular ratio E_s/E_c.

The following expression is derived from the condition of equilibrium of forces,

$$\tfrac{1}{2}A_c f_c + A'_s f'_s = A_s f_s \qquad (2.8)$$

where A_c is the area of concrete in compression and A_s and A'_s are the respective area of compression steel and tension steel. By expressing all stresses in terms of ε_c by using equations 2.3 – 2.7 and simplifying, the expression can be rewritten as,

$$A_c\left(\frac{x}{2}\right) + (\alpha_c A'_s)(x - d') = (\alpha_c A_s)(d - x) \qquad (2.9)$$

Equation 2.9 shows that the neutral axis of the cracked section is located in the centroid of the equivalent section, which is obtained by replacing the areas of compression and tension steel with the respective equivalent concrete areas. The voids shown in Figure 2 indicate the area of concrete that is displaced by the compression reinforcement. In practice, these voids are usually ignored and the area of concrete in compression A_c is simply taken as bx, where b is the beam width. By substituting bx for A_c, $\varrho' bd$ for A'_s and ϱbd for A_s, Equation 2.9 becomes

$$\tfrac{1}{2}bx^2 + \alpha_e\varrho' bd(x - d') = \alpha_e\varrho' bd(x - d') = \alpha_c\varrho bd(d - x)$$

where $\varrho = A_s/bd$ and $\varrho' = A'_s/bd$. From this equation the neutral axis depth factor x/d is

$$\frac{x}{d} = -\alpha_e(\varrho + \varrho') + \sqrt{\left\{\alpha^2{}_e(\varrho + \varrho')^2 + 2\alpha_e\left(\varrho + \tfrac{d'}{d}\varrho'\right)\right\}} \qquad (2.10)$$

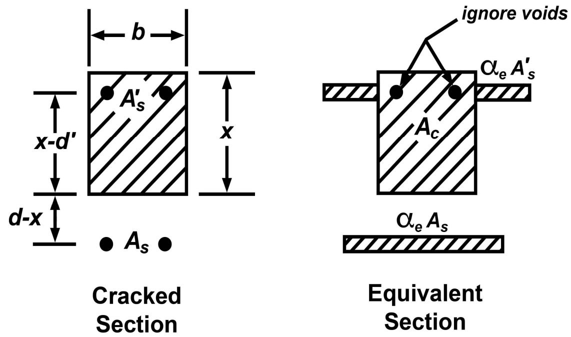

Figure 2: Equivalent section of cracked reinforced concrete section [8]

The equivalent section will now need to be incorporated into the bending formula,

$$f_{ci} = \frac{M}{I_c} x_i \qquad (2.11)$$

where f_{ci} is the concrete stress at a distance x_i from the neutral axis, M is the bending moment and I_c is the second moment of area of the cracked equivalent section. When x_i is measured into the compression zone, then f_{ci} is a compressive stress. Formulas for the maximum compressive stress in the concrete (f_c), the stress in the tension reinforcement (f_s) and that in the compression reinforcement (f'_s) can be written as follows:

$$f_c = \frac{M}{I_c} x \qquad (2.12)$$

$$f_s = \alpha_e \frac{M}{I_c}(d - x) \qquad (2.13)$$

$$f'_s = \alpha_e \frac{M}{I_c}(x - d') \qquad (2.14)$$

The second moment of area of the cracked equivalent section as shown in Figure 2 is as follows:

$$I_c = \frac{1}{3}bx^3 + \alpha_e \varrho' bd(x - d')^2 + \alpha_e \varrho bd(d - x)^2 \qquad (2.15)$$

Using the strain diagram in Figure 1 to determine the curvature of the beam at the section under consideration, results in the following formula:

$$\frac{1}{r} = \frac{\varepsilon_c}{x} \qquad (2.16)$$

By substituting $\varepsilon_c = f_c/E_c$ in Equation 2.12, we get the following curvature expression

$$\frac{1}{r} = \frac{M}{E_c I_c} \qquad (2.17)$$

<u>Case 2: The Uncracked Section</u>

If the tensile stress in a member does not exceed the tensile capacity of the concrete, then the section may be analyzed as an uncracked section, as shown in Figure 3. The effective concrete section can in this case be taken as the full section bh.

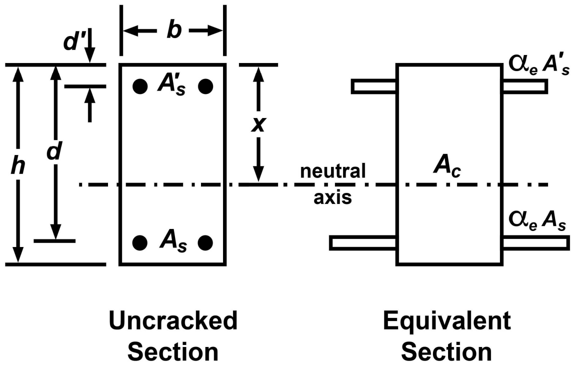

Figure 3: Equivalent section of an uncracked section [8]

The neutral axis of the uncracked section passes through the centroid of the equivalent section and can be expressed as follows:

$$A_c\left(x - \tfrac{h}{2}\right) + \alpha_e A'_s(x - d') = \alpha_e A_s(d - x) \tag{2.18}$$

Where A_c is the entire concrete area of the cross section, i.e. bh.

The second moment of area of the uncracked section is

$$I_u = \tfrac{1}{12}bh^3 + bh\left(x - \tfrac{h}{2}\right)^2 + \alpha_e A'_s(x - d')^2 + \alpha_e A_s(d - x)^2 \tag{2.19}$$

The concrete stress f_{ci} and steel stress f_{si} at any distance from the neutral axis x_i are given by

$$f_{ci} = \tfrac{M}{I_u}x_i \tag{2.20}$$

$$f_{si} = \alpha_e \tfrac{M}{I_u}x_i \tag{2.21}$$

Depending on whether x_i is measured into the compression or tension zone, the corresponding stresses are either compressive or tensile. The curvature is given as

$$\tfrac{1}{r} = \tfrac{M}{E_c I_u} \tag{2.22}$$

Case 3: The Partially Cracked Section

The partially cracked section retains some tensile stress in the tension zone, as shown in the stress distribution diagram in Figure 4. Essentially this means that some tensile capacity still exists in the concrete around the tension reinforcement, which is also known as tension stiffening. The concrete tensile stress f_{ct} is a set value, which is not affected by varying values of applied moments. The value of f_{ct} is specified in the relevant design code.

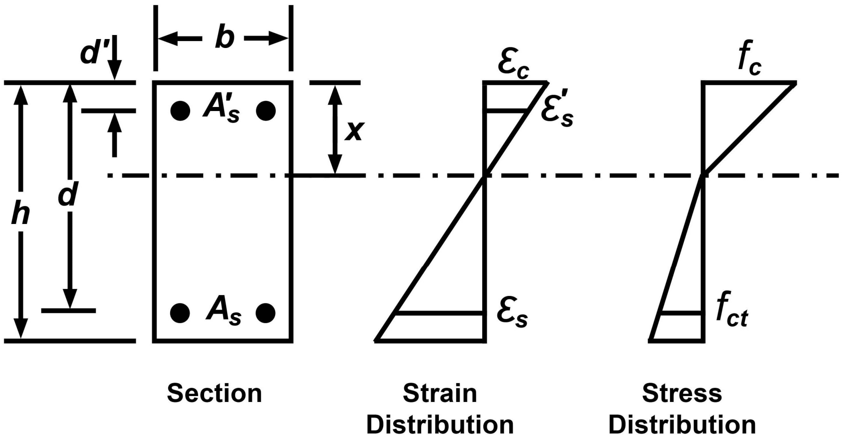

Figure 4: Partially cracked section stress and strain distribution diagrams [8]

The neutral axis depth is calculated as per the cracked section. This is a simplified assumption that is made for practical reasons. In reality, the x-axis would have to be computed by equating the tensile and compressive forces.[8] In a partially cracked section, a part of the applied moment is resisted by the concrete in tension, which is given by the following expression:

$$M_t = \frac{1}{3}\frac{b(h-x)^3}{(d-x)} f_{ct} \tag{2.23}$$

The concrete in compression and the forces in the reinforcement resist the net moment.

$$M_{net} = M - \frac{1}{3}\frac{b(h-x)^3}{(d-x)} f_{ct} \tag{2.24}$$

The formulas for the stresses therefore are

$$f_c = \frac{M_{net}}{I_c} x \tag{2.25}$$

$$f_s = \alpha_e \frac{M_{net}}{I_c}(d-x) \tag{2.26}$$

$$f'_s = \alpha_e \frac{M_{net}}{I_c}(x-d') \tag{2.27}$$

The curvature due to moment M_{net} is

$$\frac{1}{r} = \frac{M_{net}}{E_c I_c} \tag{2.28}$$

Where I_c is the second moment of area for a cracked section, as obtained from Equation 2.15.

Being able to correctly identify the elastic moment and second moment of area is a key aspect of being able to accurately calculate deflection. Designers therefore need to consider the state of cracking of the concrete member being analysed. The partially cracked section application is particularly important in deflection calculations as it is intended to provide a more exact assessment of the moment that is causing the deflection in the flexural member.[8] Section 3 will show how this is applied in the three design codes used in this research. Where early propping removal is to be considered, it is important that the elastic modulus can be accurately determined at the age of loading, in order to ensure that the results obtained from deflection calculations are accurate. This is not straight forward when it comes to reinforced concrete, as will be explained in the next section.

2.3 Factors Influencing the Deflection of Reinforced Concrete Members in Flexure

In order to accurately model the deflection of a RC member, one first needs to obtain the correct material properties. For concrete, this is not straightforward, due to the nature of the material. Concrete is made up of a variety of ingredients, each of which can have an effect on its properties, such as strength and elastic properties. Specifically the strength and elastic modulus increase as the concrete ages. The following sections provide an overview of the factors that affect the properties of concrete and therefore have a direct effect on deflection.

2.3.1 The Development of Strength in Concrete

A basic concrete mix consists of cement, water, and aggregate, the latter of which is usually added as a combination of fine and coarse sized particles [15 – 17]. Cement extenders and various chemical admixtures are sometimes added in order to achieve a desired concrete property, either in the fresh or hardened state [18]. When all the components have been mixed together, a chemical reaction takes place between the cement and water. This is commonly known as hydration [19]. During this process the various hydration products which make up the hardened cement paste (HCP) are formed. These are calcium silicate hydrate (C-S-H) gel, calcium hydroxide ($Ca(OH)_2$), C_4AH_{13} and C_4FH_{13}. The C-S-H gel is the largest contributor to the strength of the HCP. The wet cement paste in the concrete mix effectively locks the aggregate particles into place as it hardens, thereby forming a matrix of HCP and aggregates.

The strength and permeability of the cement paste is heavily influenced by the water to cement ratio (w:c), which in turn affects the strength and durability of the concrete [5]. In fresh paste, the cement particles are dispersed in water-filled space. During the hydration process, this space is gradually taken up by hydration products until only sub-microscopic pores and capillaries remain. In order to obtain a high degree of hydration, the cement particles should be completely immersed in water to ensure that the entire surface of the particle is in contact with the water molecules. Particle size also plays a role, as the surface area increases, as the particle size decreases. A high w:c ratio can cause an increase in the number, and size, of the capillaries and pores, as the space between the cement particles become too large to be filled solely with hydration products. The result is an increase in the porosity of the HCP, as well as a weaker paste-aggregate interface. It should be noted that cement particles have a tendency to stick together in lumps, which also leaves water-filled spaces devoid of particles. Plasticising admixtures can be used to improve the distribution of the cement particles [19].

When casting concrete, it is important to compact it properly in order to expel any entrapped air [20]. Trapped air in fresh concrete leads to voids and hence a more porous concrete once it has set. Excessive voids also result in diminished concrete strength. Despite this, it is not always possible to remove all of the air using conventional construction methods.

The proportion of aggregate in a concrete mix has a direct effect on the porosity of the concrete, for a constant w:c ratio. A high aggregate content may result in a stronger concrete, since the porosity of the aggregate is minimal and therefore the overall number of voids is lower [19]. The aggregate surface texture and shape also plays an important role, whereby a rough and angular crushed aggregate will ensure a better bond and less micro-cracking than smooth gravel.

In addition to the three fundamental ingredients for concrete there is also a vast array of admixtures which have been developed to enhance different properties of concrete, both in the fresh and hardened state. Admixtures can be broadly grouped into three major groups: water-reducing, accelerating and retarding. More specialised admixtures, such as air-entraining agents and water-resisting admixtures are not discussed in this section.

Water-reducing admixtures, also referred to as dispersing admixtures, allow for a reduction in water content without compromising on the workability of a concrete mix [18]. At constant cement content, this results in increased strength and durability of the hardened concrete. They can also be used to increase the workability of a particular concrete mix, without affecting the water to cement ratio. Dispersing admixtures are adsorbed onto the surface of cement particles and work on a principle of electrostatic repulsion or steric stabilisation to cause a uniform distribution of cement particles [18]. One commonly differentiates between plasticising and superplasticising admixtures. Both work on similar principles, however superplasticisers are more powerful dispersers. The use of water-reducing admixtures can have some secondary effects, which are particularly noticeable when overdosing occurs. In such cases, there can be a retardation of the setting time as well as an increased risk of segregation and air entrainment [18, 19].

Accelerating admixtures can be grouped into set accelerating admixtures and hardening accelerating admixtures. The former will reduce the time for concrete to change from a plastic state to a hardened state, while the latter will increase the early age strength development during the first 24 hours [18]. Overdosing leads to rapid stiffening accompanied by significant heat development which can result in thermal cracking. Under normal conditions, the rate at which heat is generated during hydration is also increased, but the total heat generated will be unchanged. Accelerating admixtures are usually only used when placing concrete at reduced temperatures in order to ensure sufficient strength is gained during the first 24 hours.

Retarding admixtures, and retarding plasticising admixtures, are used to extend the setting time of fresh concrete [19]. Retarding admixtures have no effect on the water demand of the concrete whereas retarding plasticising admixtures also introduce the added benefit of increasing the fluidity of the concrete. If used correctly, the setting time can be extended by between 1.5 and 6 hours [18]. Overdosing, however, leads to an increase in the concrete's setting time and in extreme cases can lead to a loss in strength as the cement may never hydrate fully. Possible secondary effects are an increased possibility of plastic cracking and increased levels of bleeding of the fresh concrete.

In summary, concrete is made up of numerous ingredients. The properties and concentrations, or mix-proportions, of these ingredients have a direct effect on the strength development over time, as well as overall strength, elastic modulus, porosity and durability of the hardened concrete. This in turn affects the way in which concrete deforms under loading, especially due to creep and shrinkage. The use of certain admixtures can also affect the early age strength development of the concrete, which can affect the propping time. In order to accurately calculate the immediate deflection at any concrete age, design codes should provide models which incorporate all the various factors which affect deflection. Creep and shrinkage models should also incorporate the effects of different mix proportions and constituent types to enable structural designers to accurately calculate long term deflection. However, usually, the designer will not have detailed information on the exact ingredients of the concrete being used on site. The models would also become very complex. As will be shown in Section 2.3.5 and Section 3, simplified models are used by the design codes.

2.3.2 Direct Tensile Strength of Concrete and Cracking Moment

Determining the direct tensile strength of concrete is more challenging than for other materials, such as steel or timber. This is due to the brittle nature of the material, which makes it difficult to grip and align in a test rig. Failure at the grips or eccentric loading is therefore likely to occur. Indirect test methods have been developed as an alternative means of determining the tensile strength. These include the splitting test and the flexural test, which determine the tensile strength due to splitting strength, $f_{t,c}$, and flexure, f_r, (modulus of rupture) respectively. The results obtained from these test will differ due to the different stress distributions during testing. Figure 5 gives a comparison and indicates how the tensile strength increases with an increase in concrete strength [21, 22].

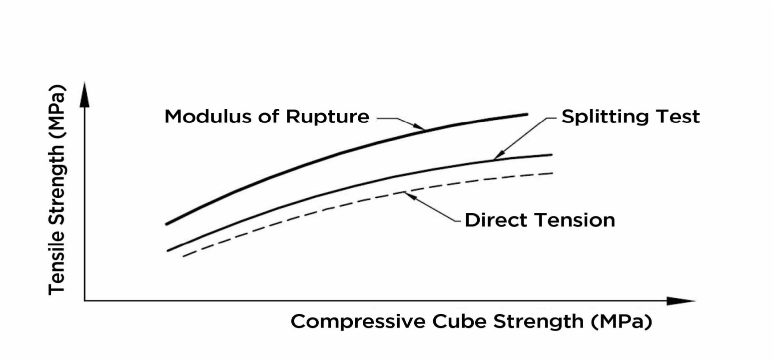

Figure 5: Relationship between direct and indirect tensile strength measurements and compressive strength [21], [22]

There is no specific or simple relationship between tensile and compressive strength, as the factors that affect the compressive strength of concrete do not affect the tensile strength to the same degree [4]. The relationship is further influenced depending on whether a cylinder or cube is used during the test.

Tensile strength is an important property, because it resists the tensile stresses in the extreme fibre during flexural loading and therefore affects deflection. Where propping times are to be reduced, structural designers need to be able to accurately assess the early age tensile strength of the concrete, in order to be able to accurately calculate the expected deflection. Design codes differ in their approach to calculating tensile strength, as will be discussed in Section 3.

2.3.3 Creep

Creep is the gradual straining of a solid body over time when exposed to a constant stress. For most structural applications, creep generally has little effect on the structural integrity of concrete members, as the deflection limits based on serviceability limit states are much smaller than the actual deflections occurring at ultimate limit state. The source of creep is in the cement paste and is largely caused by the rearrangement of water within the microstructure of the paste due to applied stresses. [23] The applied stress is carried by the hydration products in proportion to their volume fractions, while the capillary pores will only transmit very low stresses at best. This implies that the highest stress is carried by the C-S-H. Since a large fraction of the C-S-H volume is made up of micropores, the water therein can be under high stress and will want to diffuse to regions of lower stress, which are the capillary pores, in order to maintain equilibrium. As the water moves into the capillary pores, the surface forming the micropores can move closer together and the whole process can thus be described as a densification of the C-S-H through a viscoelastic response. Since C-S-H is a random array of particles, the movement of two surfaces cannot happen independently of their immediate surroundings. The movement is best described as a complex process which involves the slipping of surfaces past one another. The creep mechanism is therefore dependent on the ability of water movement by diffusion within the paste, which is a function of the pore structure and the ease of slippage of C-S-H particles, which in turn is a function of bonding.

Creep occurs at all stress levels [5]. It is affected by various intrinsic and extrinsic factors. Table 4 provides a brief overview of these factors according to [3].

Table 4: Factors affecting creep and shrinkage of concrete[5]

		Deformation		
		Elastic	**Creep**	**Shrinkage**
Intrinsic factors	**Paste factors**			
	Water:cement ratio	X	X	X
	Degree of hydration	X	X	X
	Age of paste	X	X	X
	Portland cement type	O	Possibly	Possibly
	Cement extenders	X	X	X
	Admixtures	-	Possibly	Possibly
	Temperature	O	X	X
	Moisture content	O	X	X
	Concrete factors			
	Aggregate properties and content	X	X	X
	Nature of interfacial transition zone	X	O	O
Extrinsic factors	Level of applied stress	X	X	-
	Duration of load	-	X	-
	Curing	O	X	O
	Age at loading	X	X	-
	Relative humidity and temperature	-	X	X
	Rate and time of drying	O	X	X
	Member geometry and size	O	X	X
X = Significant effect, should usually be considered O = Minor effect, can usually be neglected - = No significant effect		Load dependent	Load and environment dependent	Environment dependent

Intrinsic Factors

As mentioned in Section 2.3.1, the w:c ratio plays a major part in controlling the strength, stiffness and permeability of cement paste. A stronger paste and a paste with a denser micro structure experiences less creep strain. It can therefore be said that a decrease in w:c indirectly relates to a decrease in creep. Figure 6 shows the effects of w:c ratio on creep of cement paste.

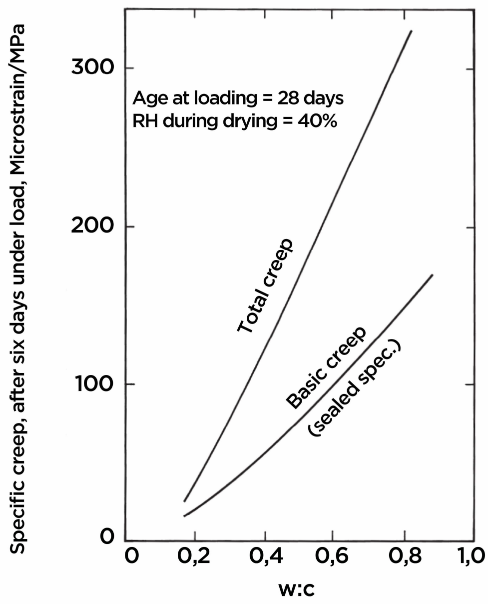

Figure 6: Effects of w:c on creep of cement paste [5]

Since creep is related to the movement of water, the moisture content also has an effect in that concrete pastes with lower moisture contents at the time of loading will creep less than a saturated paste [5]. The increase in creep is due to the drying creep component. This means that a member that has been allowed to dry before loading will experience less creep than a member that is only allowed to dry once loading commences. In the case of early propping and formwork removal, the concrete will in most cases not have had enough time to dry out.

The cement type and cement extenders seem to influence creep due to their effect on strength development of the concrete, especially at early ages [5]. Concrete made from rapid-hardening cement, for example, will tend to creep less than concrete made from ordinary Portland cement. The cement composition also affects creep, with higher C_3A contents or low C_3S contents resulting in higher creep. Cement extenders can cause an increase in creep, which is most notable during the early age of the concrete. This can be attributed to a reduced cement content which results in a microstructure that is less dense at early ages due to a reduced rate of hydration.

Admixtures can have varying degrees of effect on creep in concrete. These depend on the type of admixture and how it affects the strength development of concrete, as explained in Section 2.3.1.

Aggregates usually do not experience any creep effects at the stress levels they are exposed to in normal concrete. This is especially the case for normal-density aggregates that have been derived from hard gravels and crushed rock. In a concrete mix, the aggregate reduces the paste concentration and acts like a restraint for the paste. The aggregate volume concentration, grading and elastic modulus can therefore affect the creep of concrete [19].

Extrinsic Factors

The volume to surface area ratio, which relates to the size and geometry of a member, affects the magnitude of creep, for members loaded under drying conditions. Under drying conditions, a portion of the deformation consists of shrinkage, while the rest is due to creep. This means that the creep can be separated into two components: basic creep and drying creep. Members with large volume to surface area ratios will experience less drying creep and hence less ultimate creep [5].

Relative humidity is an important factor to consider. Under drying conditions, concrete exposed to low relative humidity tends to creep more. This is especially the case for concrete members which are allowed to dry for the first time under loading [5, 19]. The temperature also plays a role in the increased creep, especially for temperature ranges up to 50°C. The increase in creep can be attributed, to the rapid expulsion of evaporable water. At elevated temperatures in excess of 150°C, creep strain also increases due to the alteration of hydration products. However, such temperatures would not usually occur under normal loading conditions.

Creep occurs over long periods of time and is therefore also affected by load duration. Depending on the member size, 50% of the 20 year creep of a loaded member occurs within two to six months and 80% within the first two years [5]. Typically, only permanent loads are considered to affect creep. The type of permanent load is dependent on the type of structure, but will at the very least be the self weight of the structural member itself. In most cases, additional permanent loads, such as partitioning and surface finishes, will gradually be applied as the construction of the structure progresses. Design codes also provide recommended percentages of the imposed short duration load cases which may be considered to be quasi permanent and therefore also affect the long term deflection.

Concrete which has been adequately cured increases in strength over prolonged periods of time. The gain in strength can be attributed to the increased bond strength within the C-S-H. Concrete members that are loaded at a later age therefore tend to creep less. [5]. The reduction in creep at later ages of loading can be attributed to an increased degree of hydration, which results in a denser microstructure, which provides more restraint against the applied stresses.

Specific creep is the creep strain per unit stress, which occurs at low stress levels of up to 40% of ultimate strength, where the relationship between applied stress and creep is linear [5]. At this level, no significant compressive micro-cracking occurs in the concrete. Higher stresses result in additional creep strains as cracking takes place.

The following diagram shows the variation of creep strain with time for a flexural RC member.

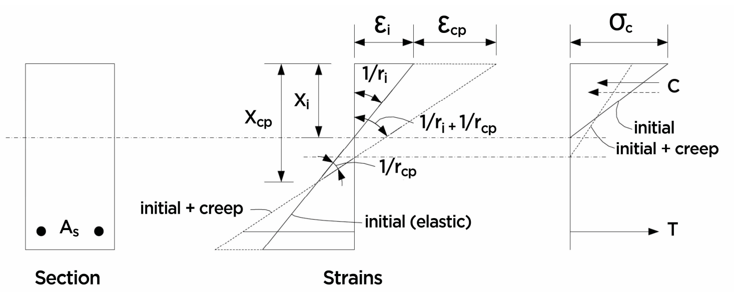

Figure 7: Creep curvature in a flexural member [12]

The creep coefficient, C_t, provides a measure of creep in concrete at any given time and is defined as the ratio of the creep strain, ε_{cp}, to the initial elastic strain, ε_i. In order to predict the maximum deflection of a flexural member due to creep, one requires the maximum value of C_t. This is called the ultimate creep coefficient.

The distribution of creep strains across the depth of the section of a flexural member is non-uniform. Instead it follows a practically linear variation similar to that produced by the applied loading. Due to this linear variation, the creep strains result in a creep curvature, $1/r_{cp}$, which is additive to the initial elastic curvature, $1/r_i$. It is also similar in effect to the shrinkage curvature, $1/r_{cs}$, which will be described in Section 2.3.4.

Although creep is mostly related to increased strain under compression, there is also a small increase in the tensile strain in the steel, as shown in Figure 7 above. Due to the creep in the concrete, there is a slight increase in the depth of the neutral axis, which results in a reduction in the internal lever arm. In order to maintain static equilibrium with the applied moment, the stress in the steel is increased, which results in the increase in steel strain.

The following expression for the creep coefficient can be derived from Figure 7 above, where it may be assumed that the creep curvature is proportional to the initial elastic curvature:[12]

$$\frac{1/r_{cp}}{1/r_i} = \frac{\varepsilon_{cp}/x_{cp}}{\varepsilon_i/x_i} = k_r C_t \tag{2.28}$$

Where $k_r = x_i/x_{cp}$ is the ratio of the initial neutral axis depth (x_i) to the neutral axis depth due to creep (x_{cp}), and $C_t = \varepsilon_{cp}/\varepsilon_i$ is the creep coefficient. Since x_i is smaller than x_{cp}, the coefficient k_r is less than unity. In singly reinforced members $k_r \approx 0.85$ [11]. This is a function of material specific creep characteristics. The presence of compression reinforcement can cause a reduction in the creep strain ε_{cp}, and hence a reduction in the creep curvature. This is due to the additional restraint that the reinforcement creates within the compression zone of the section.

Since it may be assumed that the creep curvature along the span of the flexural member is identical to the variation of $1/r_i$, the following expression can be adopted:

$$\frac{\Delta_{cp}}{\Delta_i} = \frac{1/r_{cp}}{1/r_i} = k_r C_t \tag{2.29}$$

Where Δ_i is the maximum initial elastic deflection and Δ_{cp} is the additional deflection due to creep. When estimating the maximum deflection due to creep, the ultimate creep coefficient should be used instead of C_t, as shown in Equation 2.30

$$\Delta_{cp} = (k_r \times \Phi)\Delta_i \tag{2.30}$$

In this case, Δ_i is to be taken as the initial displacement due to the quasi permanent applied loads. Some codes give guidelines to the load combinations consisting of permanent and imposed load that should be considered [24]. Even though imposed loads are temporary, it is possible that they have caused a reduction in the flexural stiffness due to cracking. The stiffness used to calculate Δ_i should therefore always consider the total permanent and imposed loads.

When calculating the long-term deflection due to permanent load and creep, an effective modulus of elasticity of concrete, (E_{eff}), may be used. This is aimed at accounting for the long-term effects of creep within the stiffness variable. To formulate E_{eff}, it is assumed that the total strain in concrete ε_{i+cp}(initial strain + creep strain) is directly proportional to the stress σ_i induced by the permanent loads [12].

$$\varepsilon_{i+cp} = \varepsilon_i + \varepsilon_{cp} = \varepsilon_i(1+\Phi)$$

$$\text{Then } E_{eff} = \frac{\sigma_i}{\varepsilon_{i+cp}} = \frac{\sigma_i}{\varepsilon_i} \times \frac{1}{1+\Phi} = \frac{E_c}{1+\Phi} \tag{2.31}$$

Where $E_c = \sigma_i/\varepsilon_i$

A graphical explanation of this principle is shown in Figure 8.

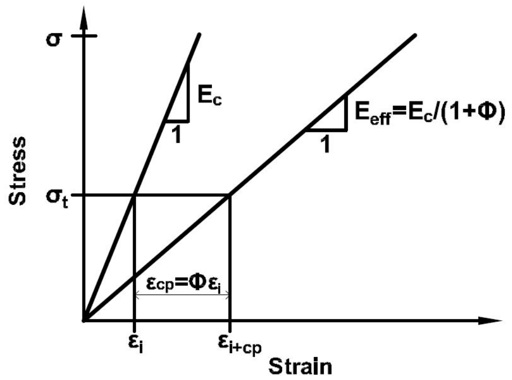

Figure 8**: Effective modulus of elasticity under creep** [12]

In summary, creep is affected by the density and bond strength of the microstructure of the concrete. The bond strength and density of the microstructure develops over time, as the freshly cast concrete sets and the hydration process takes place. The rate at which this occurs is affected by a multitude of intrinsic and extrinsic factors, most of which also affect the compressive strength of the concrete. In order to accurately model creep, these factors need to be considered. However, if all factors were to be considered, the creep models would be rather complex, and as was mentioned before, in most cases the structural designer will not have access to all this information. Section 2.3.5 shows the parameters considered by various codes.

2.3.4 Shrinkage

Shrinkage occurs due to a loss of water from the concrete. The two most important forms of shrinkage which occur in RC members are drying shrinkage and autogenous shrinkage. Newly cast concrete can also experience plastic shrinkage which can lead to significant cracking during the setting process. The loss of water during the setting period generates capillary tension in the pore water, which in turn leads to cracking in the plastic concrete, as it is not capable of resisting the forces being generated. Proper curing techniques are imperative in preventing excessive cracking.

Drying shrinkage is related to the loss of moisture to the environment, which can occur when the concrete formwork is removed and the structural member is exposed to the elements [25].

Exposed surfaces experience more shrinkage than the interior of the concrete member, which creates a non-linear strain across the cross-section of the element. The resulting internal stresses are tensile at the exposed surfaces and compressive in the interior of the section. Surface cracks are formed if the tensile stresses at the surface exceed the tensile strength of the concrete, which is likely to happen if the concrete has not matured sufficiently by the time drying commences. Appropriate curing techniques, such as wet curing, can significantly reduce the risk of cracking [25].

Autogenous shrinkage is the consumption of water during the hydration process. This form of shrinkage is minimal in normal strength concrete, but can increase significantly in high strength concrete [19].

Shrinkage is a type of deformation, which affects the overall deflection of concrete members and should therefore be taken into consideration by structural designers. Shrinkage can also cause cracking, which affects serviceability of the concrete member.

Shrinkage originates in the cement paste, and is affected by various ingredients in the concrete mixture, as well as environmental factors. These are similar to the factors that affect creep.

Shrinkage induces compressive stress in the reinforcement of RC members, which is balanced by the tensile stress in the concrete. In flexural members, where the reinforcement is unsymmetrical, the strain across the depth of the section is unsymmetrical which leads to curvature that causes deflections in the same direction as experienced due to the applied loads [11, 26].

The extent of the effects of shrinkage stress on cracking of concrete is unclear. The reason for this is twofold. Firstly, shrinkage can occur before any imposed loads are applied and subsequent cracking occurs. Secondly, additional curvature due to shrinkage effects is only nominally affected by a cracked section, because shrinkage shortening occurs in the compression zone, whereas cracks due to loading and deflection occur in the tension zone. Shrinkage forces can be regarded as being axial in nature and are therefore not resisted in the same way by a cracked section [11].

Figure 9 shows the effects of shrinkage on flexural deformation. Shrinkage is independent of the load level, and there is a parallel shift of the moment curvature diagram in both the cracked and uncracked sections.

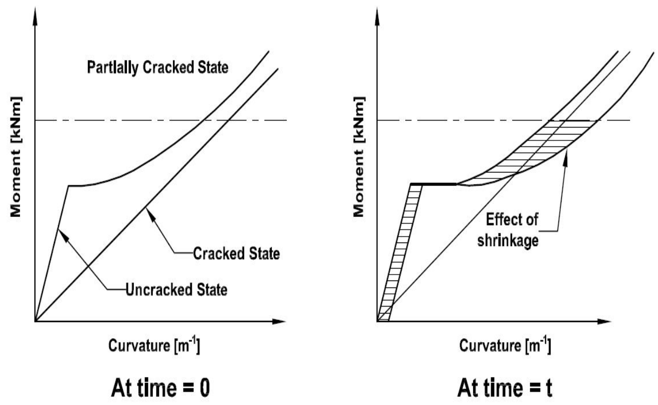

Figure 9: Effect of shrinkage on flexural deformation [26]

Shrinkage deformations have been found to be significantly different in cracked and uncracked sections [26]. It is therefore necessary to account for this difference when estimating the long-term deflection due to shrinkage. It will be shown that only the Eurocode method accounts for the difference between the cracked and uncracked states.

Compression steel can significantly reduce deflections caused by shrinkage effects. The restraint created by the compression steel, together with the restraint of the tension steel causes a reduction in the resultant tension in the concrete and with it a reduction of the shrinkage curvature.[11] In other words, an uncracked, symmetrically reinforced member will not experience any shrinkage curvature. However, shrinkage will induce a uniform stress which may cause time-depended cracking when added to the tension caused by the external loading [25].

Shrinkage in concrete continues to increase with time at a decreasing rate [25].

In summary, shrinkage can cause additional deflection in RC members. The shrinkage rate is affected by various factors, including the concrete ingredients and environmental properties. Drying shrinkage in particular needs to be considered when propping times are to be reduced, as the concrete might not have gained sufficient strength to be able to resist the shrinkage induced stresses. Shrinkage models should therefore take into account all the various factors that affect shrinkage. As is the case with creep models, the design codes only provide simplified models to calculate shrinkage deflection, as will be described in Section 2.3.5 and Section 3. It needs to be kept in mind that during the design phase, the structural designers usually do not have all the information required for a more complex shrinkage model.

2.3.5 Estimation of Creep and Shrinkage

Creep and shrinkage can be estimated at various levels of accuracy, depending on the quantity and quality of the available data. The level of accuracy required is usually dependent on the type of structure. For deflection sensitive structures, estimates are based on extensive laboratory testing as well as mathematical and computer analysis. For approximate calculations, an estimate can be made on the basis of a few parameters, e.g. relative humidity, age of concrete, member dimensions, concrete strength and general environmental conditions. At the design stage, these are often the only parameters available to the designer. Design codes provide empirical models for estimating creep and shrinkage strains. These models do not account for all of the factors that influence the creep deformation and are therefore inherently inaccurate.

Creep Models

Various creep models have been developed over the years such as the effective modulus method and the age adjusted effective modulus method and the rate-of-creep method. The effective modulus method is the most widely used, but is not particularly accurate [27, 28].

Code based creep prediction models mostly express the estimated creep strain in terms of a creep coefficient. The creep coefficient may be used together with the initial elastic modulus of the concrete, to calculate an effective modulus, which can then be used to calculate the long-term creep deflection [29]. The initial elastic modulus in these models is a predicted value which is derived from empirical equations, which differ depending on the code used. The accuracy of the predicted elastic modulus and the creep coefficient therefore greatly affects the accuracy of the estimated deflection due to creep effects [29]. Each model uses input data derived from various parameters which make an allowance for various factors that affect creep. The type of parameters and the number of parameters used in the calculations differ between models, as shown in the summary given in Table 5. The effect that each of these factors has on the predicted creep strain varies. Some models are also more sensitivity to certain factors than others. In most models, the relative humidity and the compressive strength of the concrete seem to have the biggest influence on the long term creep effects [30]. The elastic modulus was found to also affect the creep deformation. However the effects of the elastic modulus tend to reduce as the age of loading increases. This is as expected, as the elastic modulus increases rapidly at the early ages of up to seven days, after which is slows down drastically.[31] At higher ages of loading the value of the elastic modulus therefore tends to not change significantly and as a result affects the deformation to a lesser degree than at early ages of loading.

The accuracy of various code-based creep prediction models has been researched. Based on available data by various researchers, the model from the International Union of Testing and Research Laboratories for Materials and Structures (RILEM) Model B3 is considered to give the best results for creep prediction [29, 32]. It is also the most comprehensive model which takes into account the greatest number of factors that affect creep. However, the research also noted that no correlation could be found between the accuracy of the model and the number of factors being considered by the models. SANS 10100-1 uses the BS 8110 (1997) method for predicting creep, but has incorporated specific values for the elastic modulus of the aggregate type [29]. Both the SANS and ACI models seem to underestimate the creep stains.[29] The models for the SANS, ACI and Eurocode will be described in more detail in Section 3.

Table 5: Summary of factors accounted for by different creep prediction methods [32, 33]

METHOD		**SABS 0100 (1992)**	**EC 2 (2004)**	**ACI 209 (1992)**	**AS 3600 (1988)**	**CEB-FIB (1970)**	**CEB-FIB (1978)**	**CEB-FIB (1990)**	**RILEM Model B3 (1995)**
Intrinsic Factors	Aggregate Type	X							
	A/C Ratio								X
	Air Content			X					
	Cement Content					X			X
	Cement Type		X			X	X	X	X
	Concrete Density			X	X				
	Fine/Total Aggregate Ratio (Mass)			X					
	Slump			X					
	W/C Ratio					X			X
	Water Content								X
Extrinsic Factors	Age at First Loading	X	X	X	X	X	X	X	X
	Age of Sample								X
	Applied Stress	X	X	X	X	X	X	X	X
	Characteristic Strength at Loading	X	X						
	Cross-section Shape								X
	Curing Conditions								X
	Compressive Strength at 28 Days		X	X	X	X	X	X	X
	Duration of Load		X	X	X	X	X	X	X
	Effective Thickness	X	X	X	X	X	X	X	X
	Elastic Modulus at Age of Loading	X	X						X
	Elastic Modulus at 28 Days	X		X	X	X	X	X	X
	Relative Humidity	X	X	X	X	X	X	X	X
	Temperature							X	X
	Time Drying Commences								X

Shrinkage Models

Design codes provide empirically derived shrinkage models which can be used to calculate the estimated shrinkage strain in RC members.[14, 34, 35] These models are similar to the creep models in that they make use of input data derived from various parameters that affect shrinkage in concrete. The complexity of the models varies depending on the number of input parameters being considered. Table 6 provides a summary of some of the parameters that are considered in the shrinkage models of various design codes. Also included in the table is the recently proposed WITS model. A range of applicability for certain parameters is given in Table 7. In a comparison of various shrinkage models it was found that the WITS model gives the best results for South African conditions, while the ACI and the RILEM B3 model gave reasonably good predictions.[36] The SANS 10100-1 model performed poorly, which was especially noticeable at shorter drying times. [36]

Table 6: List of covariates for various code derived shrinkage models[36, 37]

Covariates	Model						
	SANS 10100-1	**EC 2 (2004)**	**ACI 209R-92**	**GL2000**	**CEB MC90-99**	**RILEM B3**	**WITS**
Concrete Raw Materials and Composition:							
Cement Type		X	*	X	X	X	X
Cement Content			X			*	X
Water Content	X					X	X
W/C Mass Ratio						*	
Air Content			X				
Sand Type							X
Stone Type							X
Stone Content							X
Sand/Total Aggregate Mass Ratio			X				
Aggregate/Cement Mass Ratio						*	
Aggregate/Binder Mass Ratio							X
Testing Conditions:							
Curing Method			*		*	X	
Age at first drying			X		*	X	
Specimen Shape						X	
Specimen Volume to Surface Area Ratio			X	X	X	X	X
Specimen Ratio of Cross-Sectional Area to Exposed Perimeter	X	X					
Temperature			*		*		X
Humidity	X	X	X	X	X	X	
Concrete Properties:							
28-day compressive strength		X		X	X	X	
28-day elastic modulus						X	
Slump			X				
*Covariate is required to assess applicability of model							

Table 7: Range of applicability of various shrinkage models[36]

Constraints	Model						
	ACI 209R-92	RILEM B3	CEB MC90-99	GL2000	SANS 10100-1	Euro-code 2	WITS
Concrete Raw Materials and Composition:							
Cement Type	Type I and III	Type I, II and III					See Appendix A
Cement Content	279-446 kg/m³	160-720 kg/m³					112-536 kg/m³
Water Content					150-230 kg/m³		160-225 kg/m³
W/C Mass Ratio		0.35-085					
Aggregate/Cement Mass Ratio		2.5-13.5					
Aggregate/Binder Mass Ratio							3.18-8.74
Sand Type							See Appendix A for list of sand types
Stone Type							See Appendix A for list of stone types
Stone Content							900-1400 kg/m³
	Model						
	ACI 209R-92	RILEM B3	CEB MC90-99	GL2000	SANS 10100-1	Euro-code 2	WITS
Testing Conditions:							
Curing Method and Time	Moist: ≥1 day or Steam: 1-3 days	Moist: ≥ 1 day or Steam	Moist ≤ 14 days	Moist: ≥ 1 day or steam			
Specimen Volume to Surface Area Ratio	1.2*exp(-0.00472*V/S) ≥ 0.2						16.5-75.0
Specimen Ratio of Cross-Sectional Area to Exposed							
Temperature	21.2-25.2°C		10-30°C				21-25°C
Humidity	40-100%	40-100%	40-100%	20-100%	20-100%	20-100%	43-72%
Concrete Properties:							
28-day compressive strength		17-70 MPa	15-120 MPa	16-82		20-90MPa	

2.3.6 Flexural Rigidity

Deflection of concrete members is a function of the applied load and the flexural rigidity of the member. The flexural rigidity (EI) of a structural member is calculated, using estimated values for Young's Modulus (E) of reinforced concrete and the moment of inertia (I) of the member. The accuracy of the deflection calculation for a given load is therefore greatly dependent on the accuracy of the values for EI.

The elastic modulus (E) of reinforced concrete is made up of a combination of the moduli for concrete E_c and steel E_s. A modular ratio is used in deflection calculations in order to accommodate the different material properties into one combined value. The value of E_s is easy to determine, due to the relative consistency of the material properties of steel that is achieved during the manufacturing process. In contrast, the E_c value is more difficult to determine due to the numerous factors that can affect the material properties of concrete, as previously discussed. The relationship of stress and strain in concrete is both non-linear and non-elastic [19]. Figure 10 below describes the stress-strain relationship of concrete. At low stresses of approximately 30-40% of ultimate strength, the stress curve is linear and the slope of the curve can be taken as the elastic modulus. This is known as the initial tangent modulus. At higher loads and under extended load durations, creep comes into effect which alters the shape of the curve to become non-linear. For practical purposes, the secant modulus is used as E_c and it is measured at 15-50% of the short-term strength.

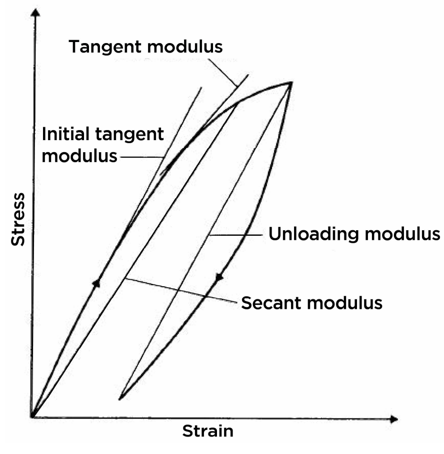

Figure 10: Stress-strain relationship of concrete [19]

The purpose of adding steel reinforcement (rebar) into concrete members is to increase the flexural stiffness of the member and decrease deflection.

Reinforcement is usually provided in the form of round bars of varying diameters and steel grades. In South Africa, rebar is available in two different steel grades: mild steel and high tensile steel. These have a tensile strength of 250 MPa and 450 MPa, respectively [38]. The available bar sizes range from 8 mm to 40 mm diameter. Since concrete is weak in tension, the reinforcing bars need to be able to resist the tensile stresses that are generated in the concrete member. The amount of rebar required depends on the magnitude of the generated tensile stresses and the strength of the rebar. The higher the tensile capacity of the steel, the lower the amount of steel required, which results in fewer bars being required. Fewer bars mean that the spacing of the bars increases. This in turn reduces the concrete's resistance to tensile stresses caused by shrinkage effects.

Where early propping removal is to be considered, structural designers need to be able to determine the elastic modulus of the concrete at the age of loading, in order to accurately calculate the expected deflection. Design codes provide empirically derived methods of calculating the elastic modulus based on the concrete strength. These methods will be further discussed in Section 3.

2.3.7 Tension Stiffening

When flexural members deflect, cracks are formed in the tension zone. These cracks are formed when the tensile capacity of the concrete is exceeded at any point along the member. For beams or slabs of uniform cross-section, this usually happens at the point of maximum moment. When a crack occurs, there is an immediate loss of stiffness in the member at the location of the crack. If the tensile stress is allowed to increase, more cracks will form which results in a loss of stiffness throughout the entire member [39]. When the member has cracked, the tensile stress is taken up by the reinforcing steel. As the member deflects and more cracks are formed, a process known as tension stiffening occurs. This is the redistribution of tensile stress into the intact concrete between cracks by the steel bars imbedded in the tension zone of the concrete. Figure 11 below shows the effects of tension stiffening on the deflection of a singly reinforced, simply-supported beam. Point A represents the point when the member first cracks due to the load exceeding the flexural tensile capacity of the concrete. If the concrete between cracks would not carry any tensile stress, then the load-deflection relationship would follow the curve of the dashed line ACD. However, if the concrete tensile stress remained equal to the concrete flexural tensile strength, the load-deflection relationship would follow the dashed line AE. The solid line of curve AB shows the actual response. The difference between the zero tension response of ACD and the actual response is due to the effects of tension stiffening.

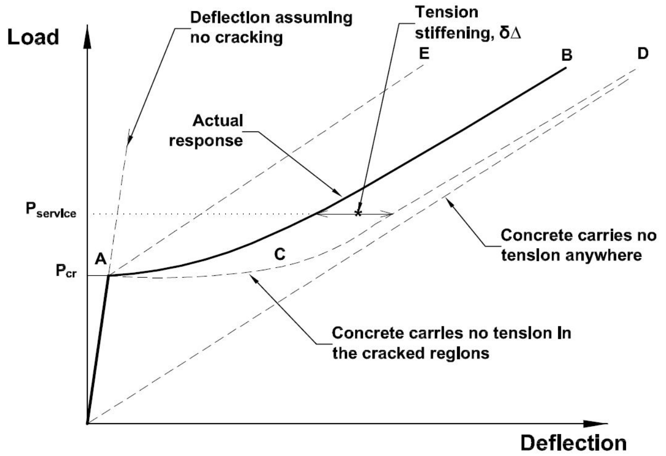

Figure 11: Typical load versus deflection relationship [39]

In order to calculate deflection, one needs to determine the moment required to cause cracking in the member and be able to model the effects of tension stiffening. This is a complex process which is further compounded by the fact that the concrete might already be cracked due to shrinkage effects.

The effects of tension stiffening on the deformation of concrete members seem to be influenced by the concentration of rebar. Heavily reinforced members are affected less than lightly reinforced members [40]. It has also been found that tension stiffening decays rapidly and will reach its long-term value of approximately half the short-term value in a period of about 20 days.

The reduction of tension stiffening can be attributed to creep, extension of internal cracks, shrinkage restraint, the formation of new surface cracks and sudden internal events [40].

The effects of creep on tension stiffening differ significantly from creep in compression, and are likely to be small and difficult to predict [40]. It is common to assume that creep in compression and tension is the same. However, for tension stiffening this is not the case. The stresses in the concrete around the rebar are tensile in nature and are much lower than the compressive stresses in the compression zone of beams and slabs. Since creep is proportional to stress, the creep will also be relatively small. The tensile stresses in the concrete around the rebar are at their highest at initial cracking and reduce as further cracking develops due to strain. Depending on the rate of loading, the creep might even be in recovery mode, which would make it very small or even negative.

A further reduction of the tension stiffening effects is due to the formation of internal cracking in the concrete around the rebar [41]. Research has shown that internal cracks develop from the ribs on deformed bars under tension. Over time, these cracks could lengthen and decrease the stiffness of the bond between the bars and the concrete, resulting in a decrease in stress being transferred into the concrete.

The major cause for the decrease of tension stiffening over time seems to be due to an increase in cumulative damage to the concrete member in the form of cracks, both internal and at the surface. The increase in accumulative damage is due to a reduction in the tensile strength of the loaded concrete with time [40, 42].

When compared to the initial deflection, the additional change in deformation due to a loss in tension stiffening is relatively small.

2.3.8 Modulus of Rupture

Some design codes (ACI, SANS) use the modulus of rupture, f_r, to calculate the cracking moment, M_{cr}. If the applied bending moment exceeds the cracking moment, then the member may be considered to be cracked or partially cracked and this may influence the magnitude of the initial deflection.

As previously discussed, the effects of shrinkage under drying conditions cause tensile stresses in a concrete member. If the member is restrained, there is a build-up of these tensile stresses. Possible sources for shrinkage restraint include embedded reinforcing bars, stiff supporting elements, adjacent members cast at different times and nonlinear gradient of shrinkage strains over the thickness of the member. The tensile stresses develop over time, as shown in Figure 12. The combined effect of stresses caused by shrinkage restraint and those developed due to applied loads results in the formation of cracks when the tensile capacity of the concrete is exceeded. The net effect is a reduction in flexural stiffness [43].

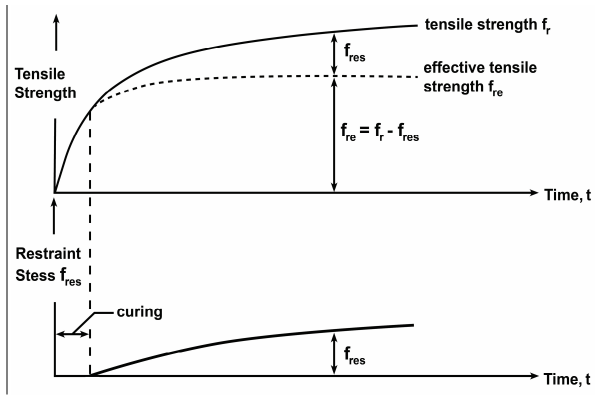

Figure 12: Development of restraint stresses in a beam [43]

Restraint stresses reduce the effective modulus of rupture of the concrete, which also decreases the cracking moment M_{cr}.

$$f_{re} = f_r - f_{res} \tag{2.32}$$

$$M_{cr}' = f_{re}\frac{I_g}{y_t} = \frac{f_{re}}{f_r}M_{cr} \tag{2.33}$$

Where f_r is the modulus of rupture of concrete, which is reduced by the restrained stress, f_{res}. The unrestrained cracking moment, M_{cr}, is based on f_r and y_t is the distance from the centroidal axis of the uncracked section to the tension face of the section.

For members that are restrained with embedded reinforcement, the time-dependent stresses that develop can be calculated as follows:

$$f_{res} = \frac{E_s\varepsilon_{cs}S_u(h-x_u)}{I_u} + \frac{A_sE_s\varepsilon_{cs}}{\left[A_c\left(1+\alpha_e(A_s/A_c)\right)\right]} \tag{2.34}$$

Where f_{res} is the tensile stress induced at the extreme tensile fibre of the concrete due to shrinkage restraint, E_s is the modulus of elasticity of steel, ε_{cs} is the free shrinkage strain, S_u is the moment of area of the reinforcement about the centroid of the uncracked section, h is the section depth, x_u is the depth to the neutral axis for the uncracked section and I_u is the moment of inertia of the uncracked section. Also, A_s is the area of tension reinforcement, A_c is the area of concrete and α_e is the modular ratio [44]. The calculation is based on the consideration of equilibrium and strain compatibility for an assumed value of free shrinkage strain [43]. The effect of the shrinkage restraint stress becomes more significant as the reinforcement ratio decreases. In heavily reinforced members, the applied service load is much greater that the cracking moment and these are therefore less affected.

2.4 Concluding Remarks

The deflection of RC members is influenced by a vast number of factors, such as section geometry, applied loading scenario, age of loading, environmental and support conditions as well as material properties. The age of loading, which is usually associated with the removal of props, can significantly affect the deflection, as concrete only develops strength over time. If propping times are to be reduced, it is important to assess the material properties at the age of loading, in order to determine when the props may be removed without causing excessive short-term and long-term deflections.

Structural designers can make use of various design codes to calculate the deflection in flexural RC members. These deflection calculation procedures are based on the beam moment-curvature relationship, which links the applied loading in the form of moments (M), material properties in the form of elastic modulus (E) and sectional properties in the form of second moment of inertia (I) to curvature. In order to accurately calculate the deflection, structural designers need to be able to obtain appropriate values for each of these input parameters. Determining the applied moments can be done through the use of analysis software and, or hand calculations, depending on the methods that are at the designer's disposal. The second moment of inertia may vary, and can be classified according to cracked, un-cracked and partially cracked sections. The state of cracking is a function of the applied load and tensile strength of the concrete at the age of loading. This means that the second moment of inertia is related to the material properties of the concrete. However, concrete is a complex material which develops strength over time. Determining its material properties is therefore not straightforward as these are influenced by the various ingredients it is comprised of, as well as various extrinsic factors. Of particular importance is the w:c ratio, aggregate content, cement type and content, curing conditions and temperature. The rate at which the strength develops is also largely dependent on these factors. Where early propping removal is to be considered, designers need to be able to accurately assess the strength and elastic modulus at the age of loading, in order to determine the effects that this has on the deflection of the member.

The deflection of flexural RC members can be separated into initial deflection and long-term deflection. Long-term deflection is the additional deflection which takes place due to creep and shrinkage effects. Creep effects are also affected by the ingredients of the concrete, as well as certain extrinsic factors such as relative humidity, curing conditions, sustained loading and age at loading. Shrinkage effects occur due to a loss of moisture from the concrete and are affected by similar intrinsic factors as creep. Design codes provide various empirical models, which may be used to calculate the creep and shrinkage effects. These models are usually simplified and cannot consider all of the factors which affect creep and shrinkage. Also, in most cases many of these factors tend to be unknown to the designer at the design stage. The accuracy of these models therefore varies considerably. The next chapter will show deflection calculation procedures for the three design codes chosen for this research, as well as their respective creep and shrinkage estimates.

3 Modelling of Deflection According to Design Standards

This section presents an overview and comparison of deflection calculation procedures for three different design codes: SANS 10100, Eurocode 2 and ACI318. In South Africa, the SANS code is the most widely used of the three. The ACI and Eurocode are also in use in the South African concrete industry and have therefore been chosen to see how they compare to the SANS code. A modelling tool containing the various code based deflection calculations was set up in an MS Excel spreadsheet. This will be used to demonstrate the differences between the codes. The following is a brief description of the MS Excel model. Thereafter follows a discussion on the short-term and long-term deflection calculation procedures of each code and how they were incorporated into the model.

3.1 Description of Deflection Modelling Tool

In order to demonstrate the different approaches taken by the three codes which are used in this comparison, it was decided to set up a modelling tool in MS Excel. Figure 13 shows the flowchart of the modelling tool.

The first tab in the spreadsheet is the Input Tab, which contains all the input parameters that are required to calculate the deflection. These include the beam geometry and section properties, the material properties and the applied loads and moments. The list of material properties contains only those parameters used by the three design codes that are being used for this investigation. The three codes are colour coded for easier reference. Each code has got three tabs which contain the various calculation procedures. The first of these tabs has been set up to calculate the immediate deflection as well as the long term deflection for different ages of loading and for different relative humidity values. The second and third tab for each code contains the calculation procedures for the creep and shrinkage coefficients, or in the case of the ACI code, the creep and shrinkage deflection calculation procedures. The first tab is linked to the input tab and the creep and shrinkage tabs. For added convenience, the first tab also displays some of the information shown on the input tab, such as the section properties, applied loading and some of the relevant material properties. Both the creep and shrinkage tabs are linked to the deflection tab and the input tab. Any changes to the input tab are therefore automatically applied to each of the tabs of the three codes.

The formulas used for the calculation of the immediate and long term deflection have been taken from the three codes. These will be described in more detail in Sections 3.2 – 3.4. The deflection results from all three codes are then plotted against each other and analysed.

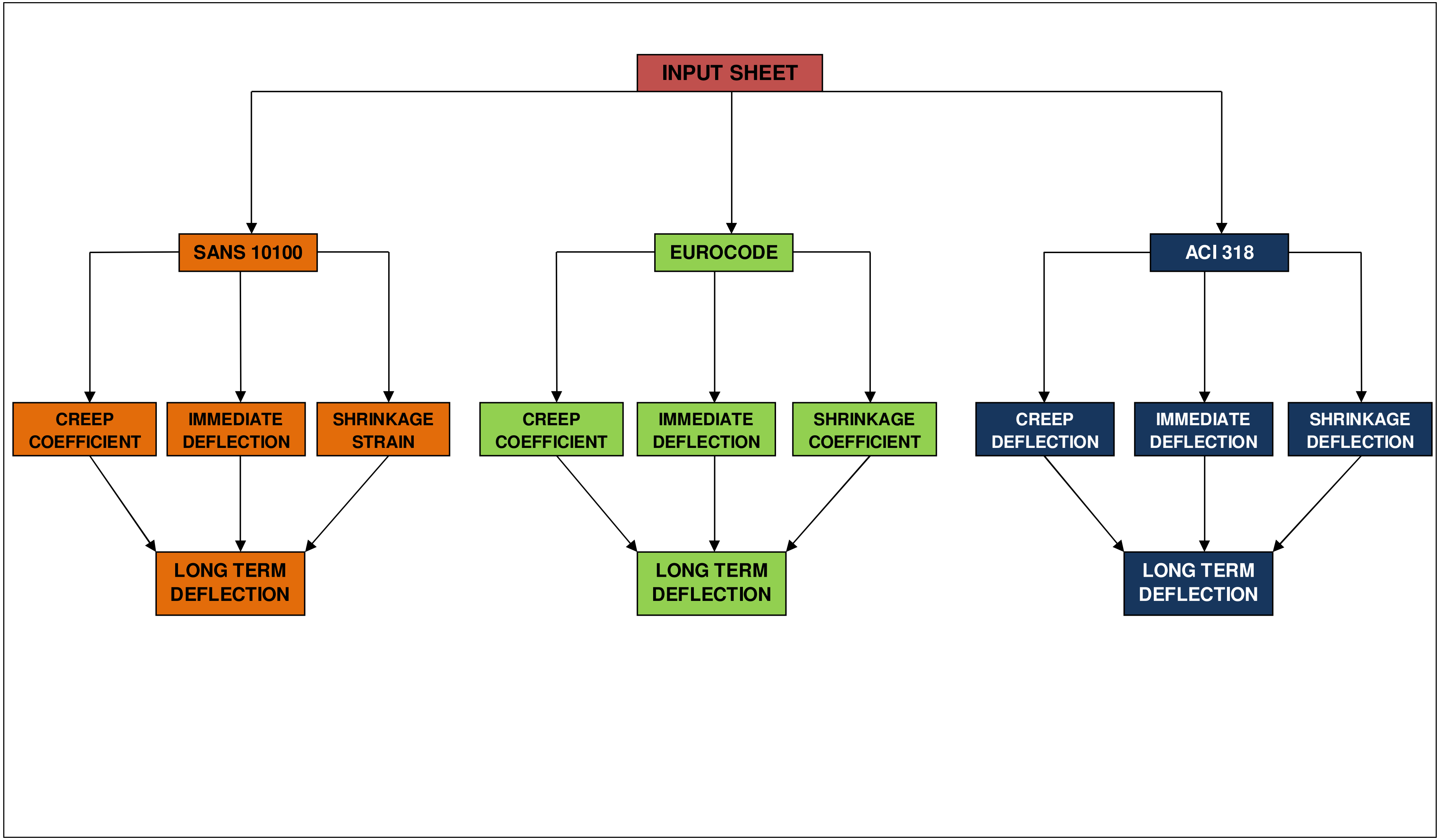

Figure 13: Flowchart of MS Excel Model used for deflection modelling

3.2 Modelling of Deflection According to SANS 10100

The SANS 10100-1 Code prescribes two methods for calculating the deflection of flexural members [14]. The first method, found in Clause A.2.3 of Annexure A of the code is based on the curvature theory that is also used in the British Standards (BS) 8110: Part 2: 1997. This method calculates the deflection by calculating the curvature at successive sections along the element and then using a numerical integration technique. The second method that can be used to calculate deflection, known as the alternative method, is given in Clause A2.4 of Annexure A of the SANS code. This method calculates the deflection due to the maximum moment caused by the permanent load acting on the member. It is a more simplified approach when compared to the first method. The deflection prediction method described in the ACI318 is similar to the alternative method, as will be highlighted later on. In order to better highlight any differences between these two code-based methods, only the alternative method will be considered for the purpose of this investigation.

3.2.1 Short-term Deflection

The immediate deflection due to the applied characteristic load is calculated using Equation 3.1, which is obtained by reformulating Equation 2.2 given in Chapter 2:

$$\Delta_i = K M_s \frac{l^2}{E_c I_e} \tag{3.1}$$

where I_e is given in Equation 3.2 below, which incorporates the degree of cracking in the element, and also accounts for tension stiffening of the concrete.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \tag{3.2}$$

The SANS code does not provide an expression to calculate I_{cr}. Instead it refers designers to some alternative literature. I_{cr} may be obtained from the BS 8007 code. Expression 3.3 gives I_{cr} in a summarised format [45]:

$$I_{cr} = \frac{ba^3}{3} + \alpha_e A_s (d - x)^2 \tag{3.3}$$

and

$$x = \alpha_e \rho \left[\sqrt{1 + \frac{2}{\alpha_e \rho}} - 1\right] d \tag{3.4}$$

The value of the cracking moment is calculated in Equation 3.5.

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{3.5}$$

Where I_g is the moment of inertia of the concrete section (ignoring reinforcement) and y_t is the distance from the centroidal axis for the uncracked concrete section (ignoring reinforcement) to the extreme fibre in tension.

The value of f_r is the modulus of rupture, such that:

$$f_r = 0.65\sqrt{f_c} \tag{3.6}$$

$$f_r = 0.30\sqrt{f_c} \tag{3.7}$$

Expression 3.6 is used for unrestrained beams and slabs; and 3.7 is used for restrained beams and slabs where pre-load cracking is likely to occur. In the above equations, the value of f_c is the cube strength of the concrete. The SANS code does not provide a method for calculating the estimated concrete strength for different concrete ages that are less than 28 days. In cases where such data is required, it is recommended that this is obtained through tests performed on the actual concrete mix that will be used [14]. Since no material testing was conducted during this research, an alternative method had to be used. Both the Eurocode and the ACI provide methods for calculating the concrete strength at different ages. These will be described in more detail in Section 3.3 and 3.4 respectively. The ACI method makes provisions for the type of curing regime used (moist or steam cured) and two types of cement used in the mix. The Eurocode on the other hand makes provisions for a wider range of cement types, while only moist cured concrete samples are considered. For the purpose of this research it was therefore decided to use the same method as given in the Eurocode.

There are various options for determining the elastic modulus (E) of the concrete, as required in Equation 3.1. The main section of Part 1 of the code contains a table with elastic modulus values which are based purely on compressive strength of the concrete. For low-density-aggregate concrete having a density between 1400 kg/m^3 and 2300 kg/m^3, the code recommends an adjustment of the estimated E values, by multiplying them by $(D_c/2300)^2$, where D_c is the density of the concrete in kg/m^3. The code also provides an alternative method, which can be used to determine the elastic modulus at any age of loading greater than 3 days. This method is based on the old British code approach and is shown in Equation 3.8.

$$E_{c,t} = E_{c,28}(0{,}4 + 0{,}6\, f_{cu,t}/f_{cu,28}) \tag{3.8}$$

This method requires the compressive strength of the concrete at age of loading ($f_{cu,t}$), the 28-day compressive strength and the 28-day elastic modulus as the input parameters. The latter may be derived from Equation 3.9, which is also based on the British code, but which has been adapted for South African aggregate types [5]. This expression can be used to calculate the effective modulus for concrete at two different age groups: early age (between three and 28 days) and later age (six months or greater). Values for K_0 and α for different aggregates can be found in Table 8.

$$E_{c,28} = K_0 + \alpha f_{cu,28} \tag{3.9}$$

The concrete strength at the age considered needs to be at least 20 MPa, when using this expression for calculating the value for $E_{c,28}$. For concrete strengths that are less than 20 MPa at age of loading, the SANS Code does not provide any method of assessing the E-value of the concrete. Since the E-values that can be obtained from Equation 3.8 are only applicable to concretes that are 3 days or older, it was decided to also limit the age of loading to 3 days in the deflection calculation spreadsheet. This will be shown in Chapter 4.

Table 8: Design values for estimating elastic modulus of concrete for ages (a) from three days to 28 days (b) at six months or older [5]

Aggregate type	Range of design values			
	(a) three to 28 days		(b) six months or older	
	K_o GPa	α GPa/MPa	K_o GPa	α GPa/MPa
Western Cape				
Granite	21	0.25	34	0.10
Greywacke (Malmesbury shale)	24	0.25	31	0.20
TM quartzite (Mossel Bay area)	23	0.25	34	0.15
KwaZulu-Natal				
Dolerite	15 - 22	0.40	29 - 39	0.15
TM quartzite	17 - 21	0.25	30	0.15
Tillite	20	0.35	29	0.20
Siltstone (KwaZulu-Natal Midlands)	21	0.15	27	0.10
Gauteng and surrounding areas				
Andesite	25 - 26	0.30	35 - 36	0.20
Dolomite	24 - 25	0.45	43 - 49	0.20
Felsite	18 - 21	0.35	29 - 31	0.20
Granite	17 - 18	0.25	25 - 31	0.10
Quartzite: Ferro	17	0.40	31	0.20
Daspoort	14	0.30	28	0.15
Reef quartzite	18 - 20	0.25	33 - 35	0.10

3.2.2 Shrinkage Deflection

Clause A.2.5 in SANS 10100-1 gives the following equation for the calculation of the shrinkage deflection:

$$\Delta_{cs} = K_{sh} k_{cs} \frac{\varepsilon_s L^2}{h} \qquad (3.10)$$

Where K_{sh} is defined for different beam types (as shown in Table 9 below), ε_s is the free shrinkage strain of the concrete and L is the effective span of the member. The value k_{cs} is defined separately for uncracked and fully cracked members, as shown in Equations 3.11 and 3.12.

$$k_{cs} = 0.7\sqrt{\rho\left(1 - \frac{\rho'}{\rho}\right)} \text{ for uncracked members} \qquad (3.11)$$

limited to $0.0 \leq k_{cs} \geq 1.0$

$$k_{cs} = 1 - \frac{\rho'}{\rho}[1 - 0.11(3 - \rho)^2] \text{ for fully cracked members} \qquad (3.12)$$

limited to $0.3 \leq k_{cs} \leq 1.0$

with $\rho = \frac{100A_s}{bd} \leq 3.0$ and $\rho' = \frac{100A_s{}'}{bd}$ limited to $\rho'/\rho \leq 1.0$

The percentage tension and compression reinforcement is defined as ρ and ρ', respectively. The respective areas for tension and compression reinforcement are given by A_s and $A_s{}'$, b is the width of the section and d is the effective depth of the section.

Table 9: Shrinkage Deflection Coefficient according to SANS10100 [14]

Beam Type	**Shrinkage Deflection Coefficient,** K_{sh}
Cantilever Beam	0.500
Simply Supported Beam	0.125
Continuous Beam End Span Interior Span	 0.086 0.063

Figure 14 below is provided in SANS 10100-1 and can be used to obtain the free shrinkage strain of plain concrete. This figure is the same as in BS 8110 (1997) and provides 6 month and 30 year drying shrinkage strains for three different effective section sizes and different relative humidity values, ranging from 20% to 100%. The shrinkage strain values for other section sizes may be interpolated as needed.

In order to automate the process of determining the shrinkage strains in the MS Excel Spreadsheet, the values for each relative humidity and effective sectional thickness were manually read off the graph and compiled into a table. A second table was then set up which contains formulas for interpolating the correct shrinkage strain values for the actual effective section thickness of the beam example being modelled.

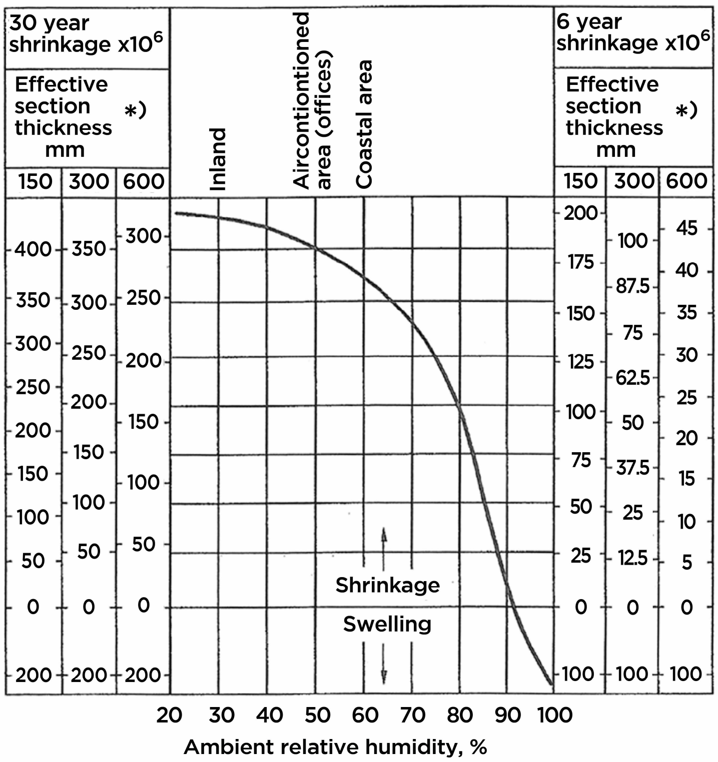

Figure 14: Shrinkage coefficient diagram for drying shrinkage of normal-density concrete as per SANS 10100 [14]

3.2.3 Long-term Deflection

The long-term deflection, Δ_l, is calculated by multiplying the initial deflection due to the permanent load, Δ_i, by a factor λ, as shown in Equation 3.13.

$$\Delta_l = \lambda\Delta_i \tag{3.13}$$

It is important to note here, that the applied moment due to permanent loads needs to be used in Equations 3.1 and 3.2 to calculate the initial deflection. The factor λ is defined in Equation 3.14.

$$\lambda = 1 + x_i\phi \quad \text{where } x_i = \frac{x}{d} \tag{3.14}$$

The ratio of the neutral axis depth to the effective depth of the cracked element is given by x_i. $\emptyset$ is the creep factor. The modular ratio used when calculating x is based on the modulus of elasticity of the concrete at the instant of loading. The term x_i represents the proportion of the concrete section which is in compression. If $x_i = 0$ then the whole section is in tension and therefore no creep would occur, as only creep in compression is considered. On the other hand if $x_i = 1$ then the whole section is in compression. The SANS code provides a diagram which can be used to read off the creep factor for different surface-to-volume ratios, and at different ages of loading and relative humidity values, as shown in Figure 15.

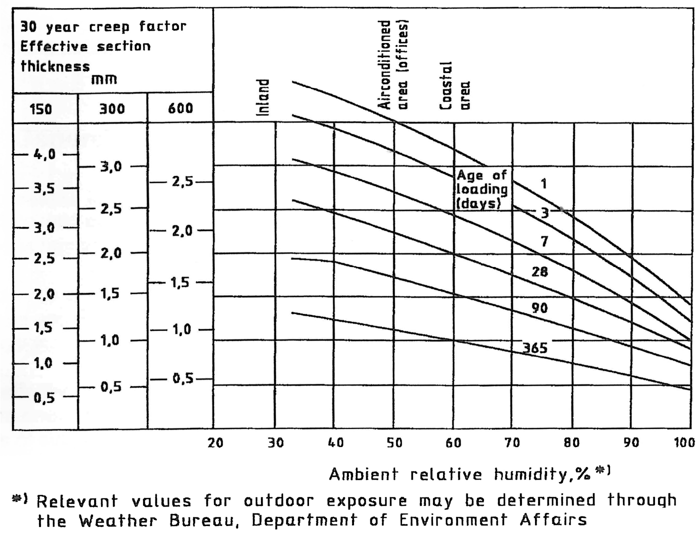

Figure 15: Creep coefficient diagram according to SANS10100

If compression reinforcement is present, then $\emptyset$ shall be substituted by $\emptyset'$ where

$$\emptyset' = \emptyset\left(1 - \frac{\rho}{2}\right) \tag{3.15}$$

And ρ is the ratio of the area of compression reinforcement to tension reinforcement.

$$\rho = A_s'/A_s \tag{3.16}$$

The data from this diagram was digitized and the plots were re-created, as shown in Figure 16. Separate curves for each of the default effective section thicknesses were plotted. A second order polynomial fit was applied to the data for the three different section thicknesses. This made it possible to interpolate for the appropriate creep coefficient based on the age of loading, RH value and section size, as given in the input tab.

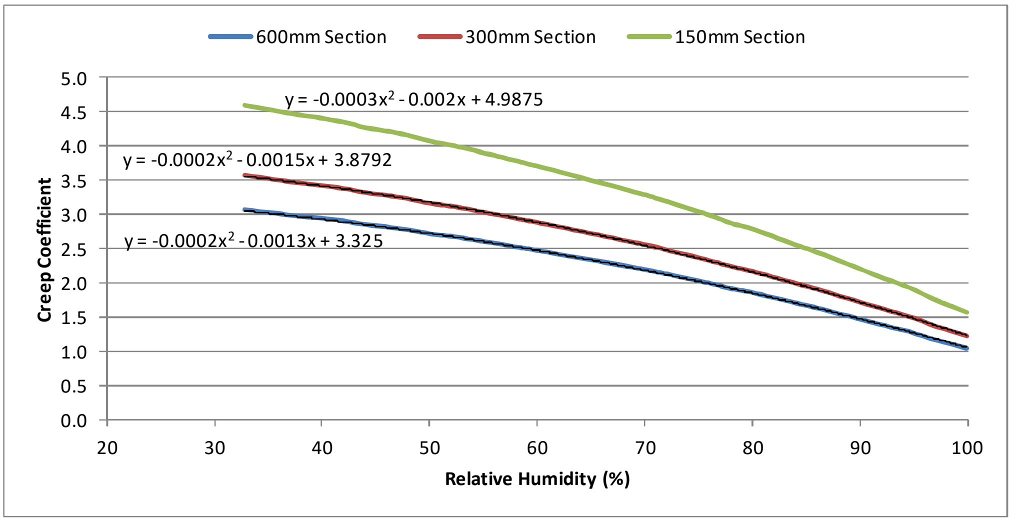

Figure 16: Digitised plot for 3 day Creep Coefficient with Polynomial Fit

The total long term deflection is calculated by adding the long-term creep deflection, shrinkage deflection and additional initial deflection. The additional initial deflection is the difference between the initial deflections calculated due to serviceability moments and permanent loads.

3.3 Modelling of Deflection According to the Eurocode

The Eurocode 2: Part1-1 (EC, 2004) provides two methods for calculating deflections [34]. Depending on the degree of accuracy required, the designer can choose between the rigorous method and the simplified method, although the former is more appropriate as it gives more realistic results.

During the rigorous method, curvature calculations are performed at frequent intervals along the length of the flexural member and at different loading stages, using a long-term elastic modulus. The loading stages that may be considered cover the period from construction to completion. This means that this method considers the effects of early age loading. When considering the loading stages, the critical loading stage at which cracking occurs needs to be identified. In cases where the deflection affects partitions or cladding, the calculations need to be repeated for the frequent loading combination and the loading stage at which the partitions or cladding is installed. The rigorous method is therefore only suitable for use with computer software, as numerous calculations and iteration need to be performed. The simplified method is similar to the rigorous method, but much less onerous. Most of the calculation steps and equations are the same as those used in the rigorous method. However, the major simplification of this method is that it does not explicitly consider the effects of early age loading. Instead, an allowance is made for it during the calculation of the cracking moment.

3.3.1 Short-term Deflection

As mentioned in Chapter 2, designers need to distinguish between a flexural member that is cracked or uncracked. If the loading does not exceed the tensile capacity of the concrete, then the member may be considered to be uncracked. Flexural members which are expected to crack can be further classified as behaving somewhere between an uncracked and a fully cracked section, for which the Eurocode provides the following expression:

$$\alpha = \zeta\alpha_{II} + (1-\zeta)\alpha_I \qquad (3.17)$$

Where α is the deformation parameter considered e.g. strain, curvature or rotation, a_I a_{II} are the values of the parameter calculated for the uncracked and fully cracked conditions respectively and ζ is the distribution coefficient which allows for tension stiffening. The following expression is given for ζ:

$$\zeta = 1 - \beta\frac{M_{cr}}{M_a}^2 \qquad (3.18)$$

Where $\zeta = 0$ for uncracked sections and β is a coefficient that takes the loading duration into account. For sustained loads or many cycles of repeated loading, $\beta = 0.5$ and for a single short-term loading, $\beta = 1.0$. M_a is the moment due to the applied serviceability load and M_{cr} is the cracking moment. If $\beta = 1.0$ then equation 3.18 reduces to

$$\zeta = 1 - \left(\frac{M_{cr}}{M_a}\right)^2 \qquad (3.19)$$

If the deformation parameter in Equation 3.17 is assumed to be curvature while ignoring tension in concrete, the equation can be rewritten using the curvature expression to produce the following expression:

$$\frac{M_a}{E_c I_e} = \zeta\left(\frac{M_a}{E_c I_{cr}}\right) + (1-\zeta)\left(\frac{M_a}{E_c I_u}\right) \qquad (3.20)$$

Equation 3.19 and 3.20 can be used to derive the effective moment of inertia for the short-term deflection prediction.

$$\frac{M_a}{E_c I_e} = \left(1-\left(\frac{M_{cr}}{M_a}\right)^2\right)\left(\frac{M_a}{E_c I_{cr}}\right) + \left(1-\left(1-\left(\frac{M_{cr}}{M_a}\right)^2\right)\right)\left(\frac{M_a}{E_c I_u}\right)$$

$$\therefore \frac{1}{I_e} = \frac{1}{I_{cr}} - \left(\frac{1}{I_{cr}} - \frac{1}{I_u}\right)\left(\frac{M_{cr}}{M_a}\right)^2$$

$$\therefore I_e = \frac{I_{cr}}{1-\left(1-\frac{I_{cr}}{I_u}\right)\left(\frac{M_{cr}}{M_a}\right)^2} \leq I_u \qquad (3.21)$$

The moment of inertia for a cracked and uncracked section is given in Equation 3.22 and 3.24 respectively.

$$I_{cr} = \frac{b{x_c}^3}{3} + \alpha_e A_s (d - x_c)^2 + (\alpha_e - 1) A_{s2} (x_c - d_2)^2 \qquad (3.22)$$

$$x_c = \left\{\begin{matrix} \left[\left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right)^2 + 2b\left(A_s d\alpha_e + A_{s2} d_2 (\alpha_e - 1)\right)\right]^{0.5} \\ -\left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right) \end{matrix}\right\} \Big/ b \qquad (3.23)$$

$$I_u = \frac{bh^3}{12} + bh\left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1)\left[A_s(d - x_u)^2 + A_{s2}(x_u - d_2)^2\right] \qquad (3.24)$$

$$x_u = \frac{\frac{bh^2}{2} + (\alpha_e - 1)(A_s d + A_{s2} d_2)}{bh + (\alpha_e - 1)(A_s + A_{s2})} \qquad (3.25)$$

The cracking moment given in Equation 3.26 is based on the uncracked properties of the section being considered. Since M_a is calculated at the critical section, M_{cr} should also be evaluated at the critical section.

$$M_{cr} = \frac{0{,}9\, f_{ctm} I_u}{h - x_u} \qquad (3.26)$$

Where f_{ctm} is given in Clause 3.1.2 in EC2 (2004) and represents the highest stress reached under concentric tensile loading, I_u and x_u depend on the properties of an uncracked section and h is the height of the section. Table 3.1 in EC2 (2004) provides the following equations for f_{ctm} in terms of the compressive cylinder strength:

$$f_{ctm} = \left(0.3 f_{cm}{}^{(2/3)}\right) \tag{3.27}$$

For $f_{cm} \leq 50/60$

$$2.12 \ln(1 + (f_{ck} + 8)/10) \tag{3.28}$$

For $f_{cm} > 50/60$

The mean compressive strength at an age t can be derived as follows:

$$f_{cm(t)} = \beta_{cc(t)} f_{cm} \tag{3.29}$$

$$\beta_{cc(t)} = exp\left\{s\left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\} \tag{3.30}$$

where cement type coefficient s = 0.2 (CEM42.5R, CEM52.5N & CEM52.5R, Class R); s = 0.25 (CEM32.5R, CEM42.5N, Class N); s = 0.38 (CEM32.5N, Class S). The code stipulates that this method can be used for concretes that have been cured according to EN12390 and at a mean temperature of 20°C. None of the other factors which might affect the strength development in concrete, as mentioned in 2.3.1, are being considered in this calculation.

The Eurocode also provides a formula for calculating the concrete tensile strength at any time, as shown in Equation 3.31:

$$f_{ctm(t)} = \left(\beta_{cc(t)}\right)^{\alpha} f_{ctm} \tag{3.31}$$

where $\alpha = 1$ for $t < 28 days$ or $\alpha = 2/3$ for $t \geq 28 days$

The concrete tensile strength can also be derived using the flexural tensile strength of the concrete. Clause 3.1.8 from EC2 (2004) can be consulted for more information about this.

The elastic modulus can be calculated as follows:

$$E_{cm(t)} = \left(f_{cm(t)}/f_{cm}\right)^{0.3} E_{cm} \tag{3.32}$$

$$f_{cm} = f_{ck} + 8 \tag{3.33}$$

where f_{ck} is the characteristic cylinder strength and E_{cm} is the elastic modulus at 28 days, as shown in Equation 3.34

$$E_{cm} = 22[(f_{ck} + 8)/10]^{0.3} \qquad (3.34)$$

An allowance should be made for the effects of aggregate type used in the concrete mix on the elastic modulus. The code recommends reducing the modulus value by 10% for limestone, 30% for sandstone, and increasing it by 20% for basalt.

The short-term deflection is then calculated using Equation 2.2.

3.3.2 Shrinkage Deflection

If shrinkage effects need to be considered, the Eurocode gives the following curvature expression:

$$\frac{1}{r_{cs}} = \varepsilon_{cs}\alpha_e \frac{S}{I} \qquad (3.35)$$

Where ε_{cs} is the free shrinkage strain, S is the first moment of area of the reinforcement about the centroid of the section, I is the moment of inertia of the section and α_e is the effective modular ratio based on the concrete's effective modulus of elasticity, E_{eff}. The free shrinkage strain is composed of the drying shrinkage, ε_{cd} and the autogenous shrinkage strain, ε_{ca}, which are expressed as follows:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$$

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \qquad (3.36)$$

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \qquad (3.37)$$

The unrestrained drying shrinkage strain, $\varepsilon_{cd,0}$ in Equation 3.36 is dependent on the concrete strength and the relative humidity. Table 3.2 in EC2 (2004) gives the value of $\varepsilon_{cd,0}$ for concrete made with CEM Class N cement. Alternatively the code provides the following expression for determining $\varepsilon_{cd,0}$ as given in Annex B of the Eurocode:

$$\varepsilon_{cd,0} = 0.85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH} \qquad (3.38)$$

and

$$\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right] \qquad (3.39)$$

Where α_{ds1} and α_{ds2} are factors for cement type and RH is the ambient relative humidity, which may range from 40% to 100%.

The coefficient k_h is depending on the notional size h_0 as shown in Table 3.3 in EC2, and $h_0 = 2A_c/u$. A_c is the concrete cross-sectional area and u is the perimeter of the cross-section which is exposed to drying.

The equation for $\beta_{ds}(t, t_s)$ is as follow

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{{h_0}^3}} \tag{3.40}$$

Where t is the age (in days) of the concrete at the moment considered and t_s is the age (in days) of the concrete at the beginning of drying shrinkage. Usually this age is taken as the duration of the curing period. In the Excel model, it was assumed that the curing period is equal to the age of loading.

The autogenous shrinkage, ε_{ca} follows from:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \tag{3.41}$$

Where $$\varepsilon_{ca}(\infty) = 2.5\left(f_c{}' - 10\right)10^{-6} \tag{3.42}$$

And $$\beta_{as}(t) = 1 - exp(-0.2t^{0.5}) \tag{3.43}$$

In the equations above, $f_c{}'$ refers to the concrete cylinder strength and t refers to the age (in days) of the concrete at the time considered.

Equation 3.35 can then be expanded to assess the shrinkage curvature, as shown in Equation 3.44.

$$\frac{1}{r_{cs}} = \zeta\varepsilon_{cs}\alpha_e\frac{S_u}{I_u} + (1 - \zeta)\varepsilon_{cs}\alpha_e\frac{S_{cr}}{I_{cr}} \tag{3.44}$$

Where S_u and S_{cr} are expressed as follows:

$$S_u = A_s(d - x_u) - A_s{}'(x_u - d')$$

$$S_{cr} = A_s(d - x_{cr}) - A_s{}'(x_{cr} - d')$$

The distribution factor, ζ, may once again be determined using Equation 3.18, but with $\beta = 0.5$ since shrinkage deformation is considered a long-term deflection process. The overall shrinkage deflection may be calculated using Equation 3.45.

$$\Delta_{cs} = K_{sh} L^2 \frac{1}{r_{cs}} \tag{3.45}$$

Where K_{sh} is the shrinkage deflection coefficient and L is the effective length of the span.

3.3.3 Long-term Deflection

When calculating the long-term deflection, there are only few modifications that need to be done to the deflection process described above. The first is the value for β, which needs to be adjusted to $\beta = 0.5$ since only sustained loads need to be considered for long-term deflection. This has the following effects on Equations 3.18 and 3.21:

$$\zeta = 1 - 0.5 \frac{M_{cr}}{M_a}^2 \tag{3.46}$$

$$I_e = \frac{I_{cr}}{1 - 0.5\left(1 - \frac{I_{cr}}{I_u}\right)\left(\frac{M_{cr}}{M_a}\right)^2} \leq I_u \tag{3.47}$$

Another modification is the use of the effective modulus of elasticity,E_{eff}, which takes creep effects into consideration.

$$E_{eff} = \frac{E_{c28}}{[1 + \varphi(\infty, t_0)]} \tag{3.48}$$

Where $\varphi(\infty, t_0)$ is the creep coefficient relevant for the load and time interval. Figure 3.1 in the code can be used to determine the 70 year creep coefficient. The Eurocode also provides an alternative method in Annex B of the code. The expression for calculating the creep coefficient is given as:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_c(t, t_0) \tag{3.49}$$

where φ_0 is the notional creep coefficient which may be estimated from:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \tag{3.50}$$

φ_{RH} is a factor which allows for the effect of relative humidity on the notional creep coefficient and may be calculated as follows:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \text{ for } f_{cm} \leq 35MPa \qquad (3.51)$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \cdot \alpha_1\right]\alpha_2 \text{ for } f_{cm} \geq 35MPa \qquad (3.52)$$

$\beta(f_{cm})$ in Equation 3.50 is a factor which allows for the effect of concrete strength and $\beta(t_0)$ is a factor which allows for the effect of concrete age at loading. These can be calculated using the following expressions:

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} \qquad (3.53)$$

$$\beta(t_0) = \frac{1}{(0.1 + {t_0}^{0.20})} \qquad (3.54)$$

The value of f_{cm} is the mean compressive 28-day strength of the concrete in MPa.

Lastly, the coefficient $\beta_c(t, t_0)$ in Equation 3.50 describes the development of creep with time after loading. The following expression may be used to estimate $\beta_c(t, t_0)$:

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)}\right]^{0.3} \qquad (3.55)$$

Where t is the age of concrete in days at the moment considered, t_0 is the age of concrete at loading and β_H is a coefficient which is dependent on the relative humidity and the notional member size, as given below:

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500 \text{ for } f_{cm} \leq 35 \qquad (3.56)$$

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3 \text{ for } f_{cm} \geq 35 \qquad (3.57)$$

Equation 3.52 and 3.57 contain coefficients $\alpha_{1/2/3}$ which consider the influence of the concrete strength in the respective expressions. The age of concrete considered was taken as 70 years in the modelling spreadsheet.

The Eurocode also makes an allowance for the effect of the cement type on the creep coefficient. This is done by modifying the age of loading t_0 according to the following expression:

$$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + {t_{0,T}}^{1.2}} + 1\right)^{\alpha} \geq 0.5 \qquad (3.58)$$

Where $t_{0,T}$ is the temperature-adjusted age of concrete at loading, as given in Expression 3.59, and α is a power which is depends on the type of cement used in the concrete. The cement types considered are Class S, N and R.

$$t_T = t_{0,T} = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i \qquad (3.59)$$

In the above expression, t_T is the temperature adjusted concrete age which replaces t in the previous expressions, $T(\Delta t_i)$ is the temperature in °C during the time period Δt_i, which is the number of days where the temperature prevails. The temperature range being considered is0-80°C. In the MS Excel model, a temperature of 23°C was used. The code also states that the coefficient of variation of the predicted creep, which has been deduced from a data bank of laboratory test results, is of the order of 20%.

The long-term deflection may then be calculated using the following expression:

$$\Delta_l = KL^2 \frac{M_a}{E_{eff} I_e} \qquad (3.60)$$

Where M_a is the maximum moment due to quasi-permanent load, L is the effective span of the member and K is the deflection coefficient that depends on the shape of the bending moment diagram. The total deflection, Δ_t, is the sum of the long-term deflection,Δ_l, and the shrinkage deflection Δ_{cs}.

3.4 Modelling of Deflection According to the American Concrete Institute (ACI) 318-11

The ACI 318-11 code provides deflection calculation methods for both the immediate and long-term deflections of RC members.[2] The following section gives an overview of each calculation method.

3.4.1 Short-term Deflection

For the immediate deflection calculation, Clause 9.5.2 of the code states that the designer may use methods and formulas such as those used to calculate elastic deflection (i.e. Equation 2.2), but must consider the effects of reinforcement and cracking on the stiffness of the member [2]. For uncracked prismatic members, the value for $E_c I_g$ may be considered to be constant along the full length of the member. A more exact calculation needs to be used for cracked sections and in cases where the depth of the member varies along the length of the span.

Stiffness values can be obtained through comprehensive analysis, or alternatively, for normal weight concrete, the immediate deflection may be computed using the modulus of elasticity of concrete, E_c, as specified in Clause 8.5.1 and effective moment of inertia, I_e, as given by the formula below. The value for I_e may not be greater than I_g.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad (3.61)$$

where M_{cr} is the cracking moment given by $M_{cr} = \frac{f_r I_g}{y_t}$

and the modulus of rupture, f_r, as $f_r = 0.623\sqrt{f_c{}'}$

In this case, the modulus of rupture is based on the cylinder compressive strength. The ACI code also provides the following expression to calculate the concrete compressive cylinder strength at different ages.

$$f_{cmt} = \left\lfloor \frac{t}{a+bt} \right\rfloor f_{cm28} \qquad (3.62)$$

Where t is the age of the concrete in days and a and b are constants which allow for the cement type and curing regime used. Values for these constants are provided in the code.

When lightweight aggregate concrete is used, f_r needs to be modified. This can be done by substituting $\sqrt{f_c{}'}$ for $1.8f_{ct}$ when the splitting tensile strength, f_{ct}, is specified. Where f_{ct} is not specified, f_r needs to be multiplied by 0.75 for all-lightweight concrete, and 0.85 for sand-lightweight concrete. In the first instance, the value of $1.8f_{ct}$ may not exceed $\sqrt{f_c{}'}$.

The effective moment of inertia formula is supposed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio $\frac{M_{cr}}{M_a}$.

The code provides the following expressions for calculating I_{cr}:

With compression steel:

$$I_{cr} = \frac{bx^3}{3} + \alpha_e A_s (d-x)^2 + (\alpha_e - 1)A'_s (x-d')^2 \qquad (3.63)$$

Where $$x = [\{2dB(1 + rd'/d) + (1+r)^2\}^{0.5} - (1+r)]/B \qquad (3.64)$$

Without compression steel:

$$I_{cr} = \frac{ba^3}{3} + \alpha_e A_s (d - x)^2 \tag{3.65}$$

Where $$x = [\{2dB + 1\}^{0.5} - 1]/B \tag{3.66}$$

In Expression 3.64 and 3.66, the values for B and r are calculated as follows:

$$B = b/(\alpha_e A_s) \tag{3.67}$$

$$r = (\alpha_e - 1)A'_s/\alpha_e A_s \tag{3.68}$$

The value of E_c may be taken as $4700\sqrt{f_c{}^i}$ for normal weight concrete or as ${w_c}^{1.5}0.043\sqrt{f_c{}'}$ for values of w_c between 1440 and 2480 kg/m³.

3.4.2 Long-term Deflection

The ACI 318 gives designers the option of obtaining the long-term deflection through detailed analysis, or by using a single expression which combines the effects of shrinkage and creep for normal and lightweight concrete. The expression, given in Equation 3.69 below, is multiplied with the immediate deflection to give the long-term deflection for a given sustained load case.

$$\lambda_\Delta = \frac{\xi}{1+50\rho'} \tag{3.69}$$

Where ρ' is the ratio of compression reinforcement at mid-span for simple and continuous spans, (and at the support for cantilevers) and ξ is a time-dependent factor for sustained loads, ranging from 3 months to 5 years or more. The values range from 1.0 to 2.0 and are specified in the code. Alternatively, Figure 17 can be used to estimate values for ξ.

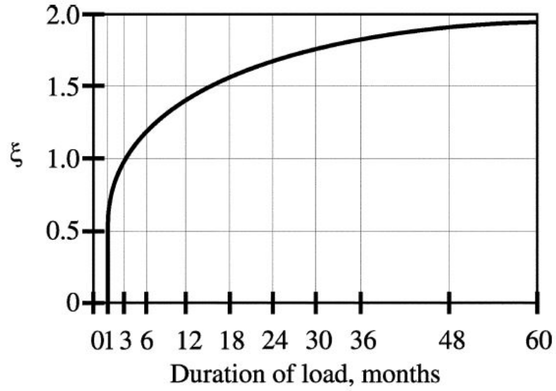

Figure 17: Multipliers for long-term deflections [2]

3.4.2.1 Deflection Due to Shrinkage Effects

If a separate and more detailed calculation of the shrinkage deflection is required, the ACI recommends the use of specialist literature, which gives the following empirical method. This method was also used in the modelling spreadsheet.

$$\frac{1}{r_{cs}} = 0.7\frac{\varepsilon_{sh}}{h}(\rho - \rho')^{1/3}\left[\frac{\rho-\rho'}{\rho}\right]^2 \qquad (3.70)$$

For $\rho - \rho' \leq 3.0\%$

Where ε_{sh} is the free shrinkage strain, h is the depth of the section and the reinforcement ratios for tension and compression steel are given by $\rho = 100A_s/bd$ and $\rho' = 100A_s'/bd$ respectively. In addition, b is the width of the section and d the effective depth. This empirical derivation proposes that the shrinkage curvature $1/r_{cs}$ is a direct function of the free shrinkage and steel content, and an inverse function of the depth of the section. The free shrinkage strain is based on recorded experimental data. [11] The following expression can be used to estimate the shrinkage strain:

$$\varepsilon_{sh}(t, t_c) = \frac{(t-t_c)^\alpha}{f+(t-t_c)^\alpha} \times \varepsilon_{shu} \qquad (3.71)$$

Where $(t - t_c)$ is the time from the end of initial curing, f and α are constants for a given member shape and size that define the time-ratio part, and ε_{shu} is the ultimate shrinkage strain. The value of the ultimate shrinkage strain under standard conditions is suggested to be 780×10^{-6}. Table 10 shows the standard conditions and variables considered by the shrinkage and creep models of the ACI code.

The ultimate shrinkage strain may be modified in cases where the standard conditions are not met or exceeded, as shown in Expression 3.72:

$$\varepsilon_{shu} = 780 \times 10^{-6}\gamma_{sh} \qquad (3.72)$$

γ_{sh} is the cumulative product of the applicable correction factors and is given in the following expression:

$$\gamma_{sh} = \gamma_{sh,tc}\gamma_{sh,RH}\gamma_{sh,vs}\gamma_{sh,s}\gamma_{sh,\Psi}\gamma_{sh,c}\gamma_{sh,\alpha} \qquad (3.73)$$

Where $\gamma_{sh,tc}$ is the initial moist curing coefficient, $\gamma_{sh,RH}$ is the ambient relative humidity coefficient, $\gamma_{sh,vs}$ is a coefficient which allows for the member size in terms of the volume-surface ratio, $\gamma_{sh,s}$ is the coefficient for the slump factor, $\gamma_{sh,\Psi}$ is a coefficient for the fine aggregate content factor, $\gamma_{sh,c}$ is the coefficient for the cement content and $\gamma_{sh,\alpha}$ is the coefficient for the air content.

The code provides expressions for each of these correction factors [35]. The relative humidity coefficient is limited to RH-values ranging from 40% to 100%.

The shrinkage deflection can then be calculated using Equation 3.74.

$$\Delta_{cs}= K_{sh} L^2 \frac{1}{r_{cs}} \tag{3.74}$$

Table 10: Factors affecting concrete creep and shrinkage and variables considered in recommended prediction method [35]

Factors			Variables considered	Standard conditions
Concrete (creep and shrinkage)	Concrete composition	Cement paste content Water-cement ratio Mixture proportions Aggregate characteristics Degrees of compaction	Type of cement	Type I and III
			Slump	70 mm
			Air content	≤6%
			Fine aggregate percentage	50%
			Cement content	279 to 446 kg/m^3
	Initial curing	Length of initial curing	Moist cured	7 days
			Steam cured	1 to 3 days
		Curing temperature	Moist cured	23.2 ± 2 °C
			Steam cured	≤100 °C
		Curing humidity	Relative humidity	≥95%
Member geometry and environment (creep and shrinkage)	Environment	Concrete temperature Concrete water content	Concrete temperature	23.2 ± 2 °C
			Ambient relative humidity	40%
	Geometry	Size and shape	Volume-surface ratio or minimum thickness	V/S = 38 mm
				150 mm
Loading (creep only)	Loading history	Concrete age at load application	Moist cured	7 days
			Steam cured	1 to 3 days
		Duration of loading period	Sustained load	Sustained load
		Duration of unloading period	-	-
		Number of load cycles	-	-
	Stress conditions	Type of stress and distribution across section	Compressive stress	Axial compression
		Stress/strength ratio	Stress/strength ratio	≤0.50

3.4.2.2 Deflection Due to Creep Effects

The effects of creep also need to be considered for long term deflection. It is suggested that the long term deflection, Δ_l, is a function of the initial deflection, Δ_i as shown in Equation 3.75.

$$\Delta_l= k_r \emptyset \Delta_i \tag{3.75}$$

Where k_r is a reduction factor and $\emptyset$ is the creep coefficient. The creep coefficient is again recorded from experimental results, examples of which have been published in various tables. Alternatively the following expression can be used to calculate the creep coefficient:

$$\phi(t,t_0) = \frac{(t-t_0)^{\Psi}}{d+(t-t_0)^{\Psi}}\phi_u \tag{3.76}$$

Where $\phi(t,t_0)$ is the creep coefficient at concrete age t due to load applied at age t_0, d and Ψ are constants for the member size and shape and ϕ_u is the ultimate creep coefficient. The ultimate creep coefficient for standard conditions is given as $\phi_u = 2.35$. However, the code provides a method for modifying the ultimate creep coefficient for other conditions, as shown in the following expression:

$$\phi_u = 2.35\gamma_c \tag{3.77}$$

Where γ_c is the cumulative product of all the correction factors, as shown in Expression 3.78:

$$\gamma_c = \gamma_{c,t0}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\Psi}\gamma_{sh,\alpha} \tag{3.78}$$

The coefficients represented in the expression are for the age of loading ($\gamma_{c,t0}$), a relative humidity factor ($\gamma_{c,RH}$), a factor for the member size in terms of volume-surface area ($\gamma_{c,vs}$), a slump factor ($\gamma_{c,s}$), a fine aggregate content factor ($\gamma_{c,\Psi}$) and an air content factor ($\gamma_{sh,\alpha}$). The code provides expressions for calculating each of these factors.

The recommended equation for the reduction factor in Expression 3.75 is given as follows:

$$k_r = \frac{0.85}{(1+50*\rho')} \tag{3.79}$$

It should be noted that the initial deflection is calculated using the permanent load, made up of permanent DL and a percentage of imposed load. This will vary depending on the type of structure and load case scenario [21].

The total deflection is calculated by adding the long-term deflection, the shrinkage deflections and the additional initial deflection due to the remaining imposed load (e.g. 70% of imposed load, where the other 30% was already added to the permanent load).

3.5 Concluding Remarks

The three codes used in the modelling spreadsheet each have got slightly different methods for calculating the immediate and long term deflection, but all are based on the beam moment-curvature relationship and elastic theory described in 2.1. Each code provides expressions for calculating the cracked and un-cracked section properties. The Eurocode provides an additional expression which may be used to calculate the degree of cracking by interpolating between an uncracked and a fully cracked section. The SANS and ACI codes make an allowance for the degree of cracking and the tension stiffening effects in the same expression that is used to calculate the effective moment of inertia. The Eurocode also makes provisions for the presence of tension and compression reinforcement when calculating the uncracked moment of inertia, while the ACI and SANS code use gross cross-sectional properties.

All three codes use simplified, empirically derived methods to calculate the various material properties required in the deflection calculation equations. The elastic modulus and tensile strength characteristics are based on the concrete compressive strength. This approach is very practical, as structural designers usually tend to have knowledge of the concrete compressive strength during the design stage. However, as was already mentioned in Chapter 2, the concrete compressive strength and the rate at which it develops over time tends to be influenced by various factors which are not specifically taken into consideration by these models. Furthermore, the factors which affect the elastic modulus do not necessarily have the same effect on the concrete strength [5]. The ACI and Eurocode provide expressions which allow the structural designer to calculate the estimated concrete strength at a particular age. Since the elastic modulus and concrete tensile strength are based on the concrete compressive strength, all factors used in the deflection calculation, which are linked to either of these values, such as the cracking moment, cracked section depth or the second moment of area of the cracked section will also be affected. This in turn will have an effect on the deflection. A comparison can therefore be drawn up to highlight the effect that early propping removal and the age of loading has on deflection. The modelling spreadsheet will be used to calculate the effects that different concrete strengths and different ages of loading have on the deflection. This will be demonstrated in Chapter 4.

The methods used by the three codes for the estimation of creep and shrinkage effects, varies considerably. In the SANS code, the shrinkage strains and creep coefficients are read off from diagrams. The diagram for the shrinkage strain only considers the relative humidity and effective section thickness, while the diagram for the creep coefficient additionally makes provisions for the age of loading. Both the ACI and Eurocode provide empirically derived expressions which allow the designer to calculate these long term effects. The long term deflection calculation procedure in the Eurocode uses an effective modulus of elasticity to allow for creep effects, while in the ACI and SANS codes the initial deflection is modified to obtain the long term deflection values. As was mentioned in section 2.33 and 2.34, creep and shrinkage are affected by various intrinsic and extrinsic factors. The only factors being considered by each of the codes, apart from the applied stress and effective thickness, is the relative humidity and in the case of creep, the age of loading. In order to compare the effects of early propping removal on the long term deflection, the modelling spreadsheet has been programmed to calculate the deflection at different ages of loading as well as relative humidity values, ranging from 40% to 90%. This will be discussed in more detail in Chapter 4.

In terms of their suitability to assessing the effects of early propping removal on deflection, the SANS code appears to be the least suitable of the three codes. This is due to the fact that the SANS code does not provide a method for calculating the estimated concrete strength at different ages. For this research, this problem is bypassed, by using the concrete strength prediction method of the Eurocode. The minimum age of concrete that can be used in the elastic modulus calculation method is 3 days. This therefore also limits the age of loading that can be used in the deflection modelling.

The ACI code does not provide a method for calculating the creep coefficient for concrete that have been cured for less than 7 days. Even though it is possible to provide curing for the concrete after loading has been applied, in practise this is difficult to achieve. From a practical perspective, the ACI code is therefore better suited for calculating the estimated deflections at ages of more than 7 days.

The Eurocode does not have the restrictions of the other two codes and therefore seems to be well suited for calculating the effects of early propping removal. However the calculation procedures for the concrete strength, creep coefficient and shrinkage strain have not been calibrated to South African cements and aggregates.

4. Comparison of Deflection Modelling Results

This chapter describes the findings of the deflection modelling, using the code based calculation procedures mentioned in Chapter 3. A typical beam type and load case was used in three different case studies to model the effects of early propping removal, relative humidity and concrete compressive strength on the long term deflection. Each case study will be discussed separately in this chapter. The three parameters mentioned were chosen, because they are common to all three code based deflection calculation procedures and are most likely to be known by the structural designer. It was also shown in Chapter 3, that the material properties are mostly derived from the concrete strength, while the long-term effects are also affected by relative humidity and age of loading. For the purpose of this investigation the age of loading was considered to coincide with the age at which the props are removed.

Figure 18 shows the typical column and beam configuration used for the deflection modelling. In this case, the structure represents a mezzanine level that is used for storage. Such structures can be commonly found in warehouses or other storage and industrial facilities with high roofs. Mezzanine levels are a convenient way of expanding the work (or storage) space of a building, without increasing the size of the footprint of the structure. The columns have been placed on a 5 m x 5 m grid. This spacing is fairly typical for this type of structure, and is a good compromise between practicality and being economical to build. The column spacing may vary, depending on the type and application of the structure. Larger column spacings are possible, but for the same load would require deeper and more heavily reinforced beams, which would drive up the construction cost. A typical beam size of 280 mm x 450 mm was used, spanning continuously over the supports. This beam size was found to be adequate for the intended load case. The floor slab is comprised of individual precast concrete slabs, spanning between beams, with a structural screed applied on top of the slabs to create a uniform surface. An in-situ reinforced concrete slab could have been used as an alternative to the precast slabs. Both systems have got their advantages and disadvantages. Cost and speed of construction are usually the overriding factors that would influence the decision on which system should be used. A detailed time-cost analysis was not included in this dissertation, as it falls outside the scope of this research.

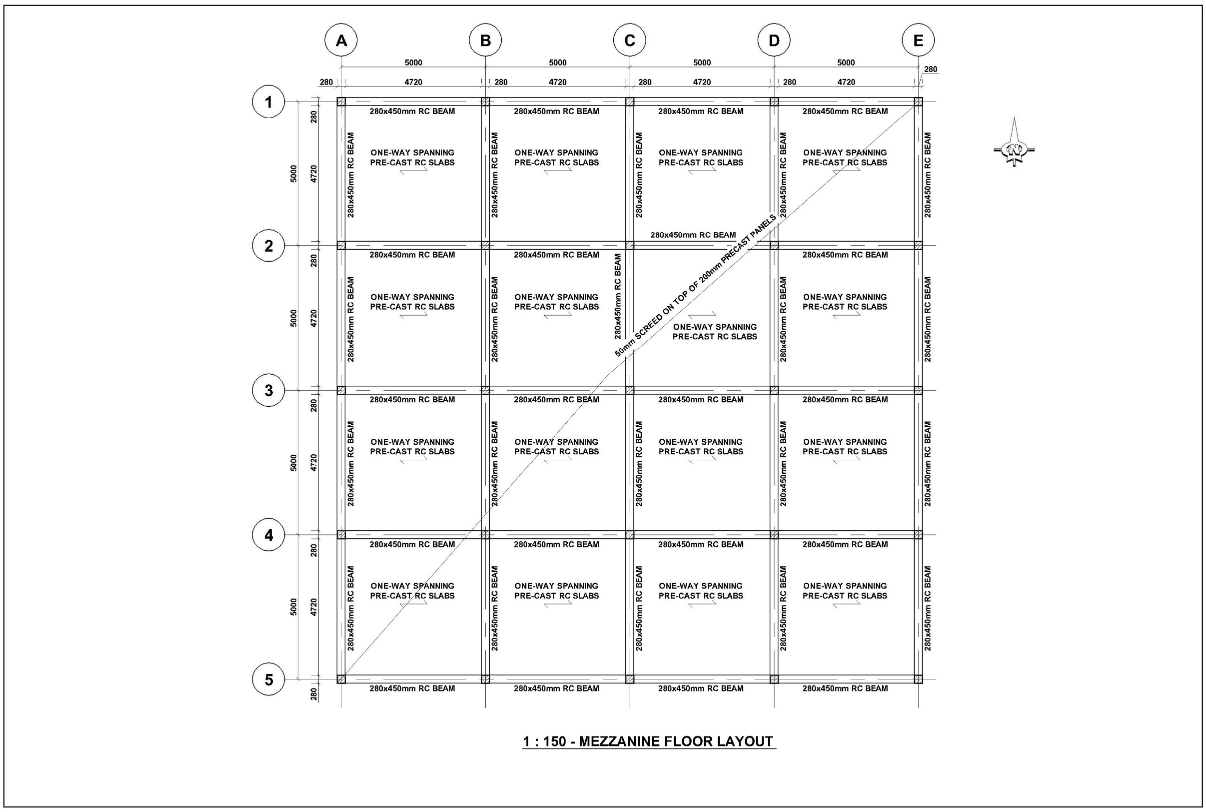

Figure 18: Layout of Mezzanine Floor Example

The mezzanine floor slab was designed to support an imposed load of 5 kN/m². This value was based on the recommended loading requirement for storage facilities, as given in SANS 10160-2.[24] The permanent loads included for the calculation of the deflection is the sum of the self weights of the beam, pre-cast slabs and structural screed, as well as 80% of the imposed load. The percentage of the imposed load which is considered to be quasi-permanent is based on the recommendations presented in the SANS 10160-1 code.[46]

The deflection limit was based on the recommended limits as set out in the SANS10160-1. For this application, the code recommends that the deflection should be limited to the lesser of the visible span/250, or 30 mm, for both the short and long-term serviceability limit state. In this case, the maximum allowable deflection was therefore set as 20 mm.

The input parameters for the material properties were based on typical concrete mix designs, as shown in Table 11 below. This table contains the full spectrum of concrete strengths that have been programmed into the modelling spreadsheet.

Table 11: Typical Concrete Mix Designs[47]

Concrete Mix Designs								
Description	**25MPa**	**30MPa**	**35MPa**	**40MPa**	**45MPa**	**50MPa**	**55MPa**	**60MPa**
Stone 19mm	975 kg	1050 kg	1065 kg	1065 kg	1075 kg	1075 kg	1075 kg	1075 kg
Dune Sand	540 kg	495 kg	485 kg	460 kg	430 kg	400 kg	400 kg	375 kg
Crusher Dust	455 kg	420 kg	410 kg	390 kg	365 kg	340 kg	340 kg	320 kg
Cement	140 kg	160 kg	170 kg	201 kg	230 kg	215 kg	225 kg	245 kg
Slag	95 kg	105 kg	115 kg	135 kg	153 kg	215 kg	225 kg	245 kg
Water	177 lt	174 lt	173 lt	172 lt	172 lt	172 lt	170 lt	172 lt
W/C	0.75	0.66	0.61	0.51	0.45	0.40	0.38	0.35
Slump	75mm	75mm	75mm	75mm	75mm	75mm	75mm	75mm

As was shown in Chapter 3, the creep and shrinkage models in the ACI code use a number of covariates, some of which are related to the concrete mix design. No information was available for the air content factor. The value for this factor was therefore taken as being equal to one.

Since the aim of this research is to determine the effects of reduced propping time on long-term deflection, the age of loading in all three case studies ranges from 3 days to a maximum of 14 days. The maximum was limited to 14 days, as this corresponds with the recommended propping time for beams as specified in SANS 2001 for hot or normal weather. In cases where the temperature is used as a covariate, such as in the creep coefficient calculation procedure in the Eurocode, this was taken as 23°C. This figure represents the approximate average of the normal weather range provided by SANS 2001. The normal weather condition represents the most likely temperature range for South African conditions.

The curing duration is limited to a maximum of 7 days. This is applicable, for example, in the ACI creep and shrinkage deflection calculations. The 7-day limit is based on a common principle used in the construction industry, whereby the formwork is stripped after a certain duration, while the propping remains in place. The sooner the formwork can be removed, the sooner it can be reused elsewhere. However, once it has been removed, the concrete is exposed to the elements and drying can commence. The curing period effectively ends at this stage.

4.1 Modelling of Long Term Deflection at Varying Ages at Loading

The first case study looks at comparing the effects that early propping removal has on the long term deflection. For this, the deflection was calculated at various ages of loading, ranging from 3 days to 14 days. As was mentioned previously, the age at loading in this case coincides with the propping removal time. Apart from the age at loading, all other factors, such as member size, span, load case and area of steel remained constant. A concrete mix for a 30 MPa strength class was used, which was found to be adequate for this application. Two versions of this mix were used for the deflection modelling. The first mix used a cement type 42.5N and the second a type 42.5R. By modelling two different cement types, a comparison could be drawn up to show how the deflection results would be affected based on the cement type used. The relative humidity was set at 60%, which coincides with the value for coastal areas as shown on the SANS creep coefficient diagram. The following results were obtained for the long term deflection.

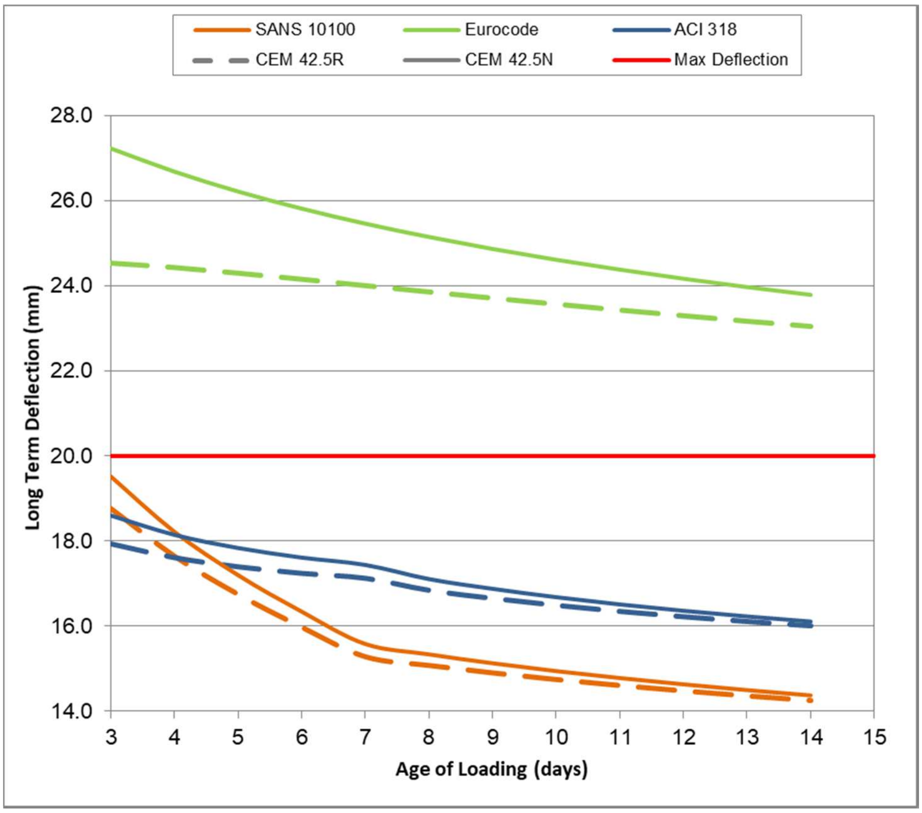

Figure 19: Effects of age at loading on long-term deflection

As can be seen in Figure 19, the deflection reduces as the age at loading increases. The rate at which the deflection reduces over time appears to be at its highest during the early ages of loading. The SANS code in particular shows a significant reduction in deflection up to the seventh day. The overall reduction in deflection can be attributed to the fact that the deflection models are based on the concrete strength, as was explained in Chapter 3. The increased rate at which the deflection reduces at early ages can be attributed to the initial rapid strength gain which tends to happen after the concrete has set and the hydration process begins, as was explained in Chapter 2. Similarly, this also explains the small distinction between the deflection values of the two cement types used, since concrete mixes which contain 42.5R cement tend to develop strength more rapidly than a mix which contains a type 42.5N cement. The Eurocode and ACI models take this into account. However the Eurocode deflection calculation method seems to be more sensitive to the cement type, especially at early ages, where the percentage difference was calculated to be up to 10%. Figure 20 shows a comparison of this strength development trend for the 30 MPa concrete mixes that were used. The data for these plots was obtained from the code based concrete strength calculation procedures. As was mentioned in Chapter 3, the method from the Eurocode was also used for the SANS code calculations, as the SANS code does not provide a method for calculating the concrete strength at different ages. The curves for the SANS and Eurocode are therefore the same. It is interesting to note that the ACI method shows a more significant difference between the predicted compressive strengths for the two cement types used.

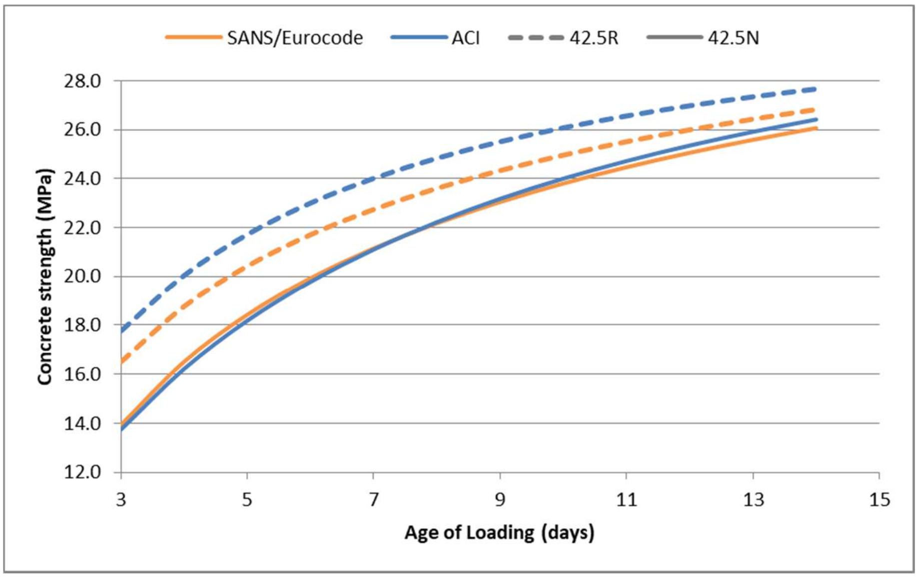

Figure 20: Comparison of concrete strength development of 30MPa Concrete Mix

The initial rapid decrease in long-term deflection observed with the SANS code deflection curve in Figure 19 can be attributed to the higher estimated creep coefficients of the SANS method during the early ages of loading. Figure 21 shows a comparison of the creep coefficients that were calculated using the code-based methods. Only one curve can be provided for the SANS code, since it does not make any provisions for the cement type being used. However, the curve of the SANS code shows a more rapid decrease of the creep coefficient value during the first 7 days, when compared to the other two codes and is similar to the trend of the long-term deflection curve in Figure 19. This seems to indicate that the SANS code method for calculating the long term deflection is more sensitive to early ages of loading than the Eurocode or ACI method. As was mentioned in Section 3.5, the time factor incorporated into the creep coefficient calculation procedure of the ACI code is limited to 7 days for moist cured concrete. This results in the constant value of the creep coefficient during this time period.

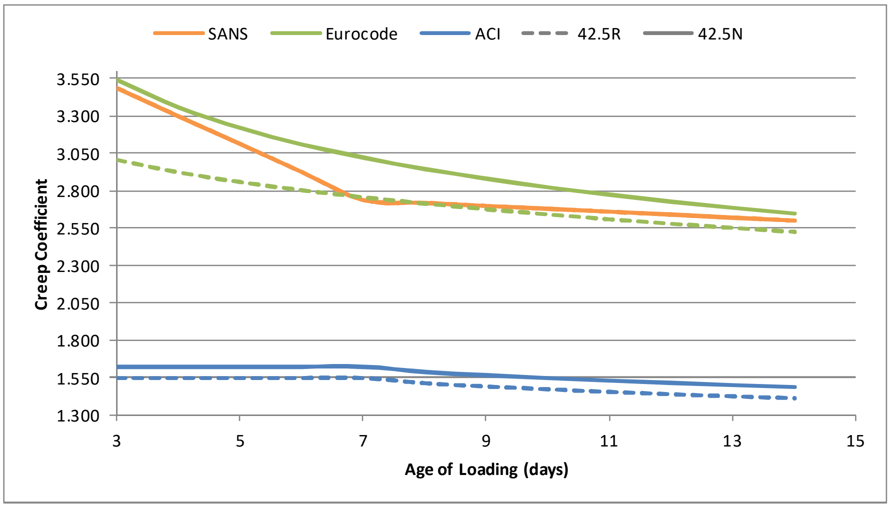

Figure 21: Comparison of creep coefficients using code based estimation methods

The trend observed in the long term deflection results of the SANS code is not present in the initial deflection results, which do not take account of creep effects, as shown in Figure 22.

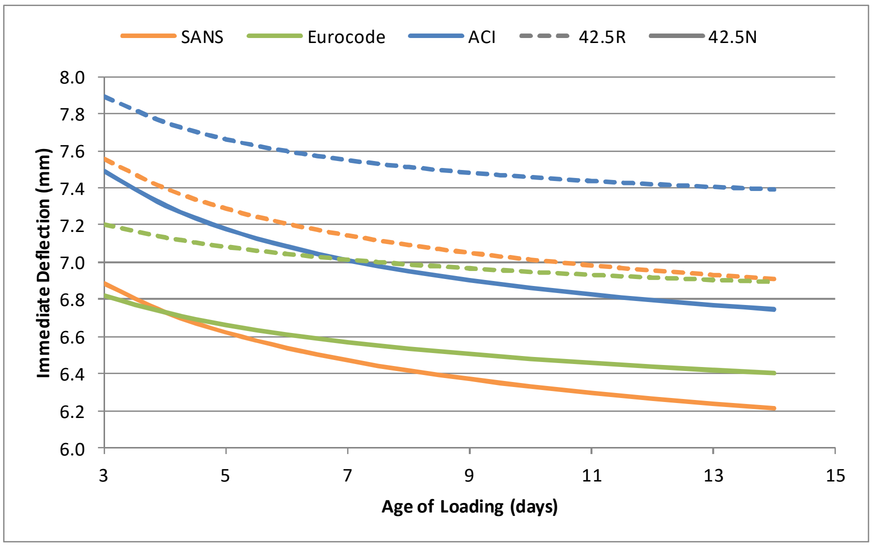

Figure 22: Initial deflection results of SANS, Eurocode and ACI models

The initial deflection results of the code based methods are also more closely matched when compared to the long term deflection results. On average, the initial deflection results only vary by between 6% – 8% between codes. In comparison, the long term deflections vary on average between 29% – 37% between codes. Even though the initial deflection results are fairly similar for all three codes, it is interesting to note that the long term deflection results obtained from the Eurocode are significantly higher than the ones for the SANS and ACI codes, as was shown in Figure 19. This can be attributed to the underestimation of the creep strains by the SANS and ACI methods, as was pointed out in 2.3.5.

Based on the deflection values given in Figure 19, the estimated long term deflections obtained from the SANS and ACI calculation procedures are lower than the recommended maximum at all ages of loading, while the Eurocode values exceed the allowable deflection even after 14 days. Based on the SANS and ACI results alone, one could motivate that a significant reduction in the propping time is possible, when compared to the recommend propping times given in SANS 2001. However, it needs to be kept in mind that the SANS and ACI codes tend to underestimate the creep effects, and the actual required propping duration might be longer.

4.2 Modelling of Long Term Deflection at Varying Relative Humidity Levels

The second case study looks at the effects of relative humidity on the long term deflection after early propping removal. As was explained in Chapter 2, the long-term effects are influenced by various factors, including relative humidity. All three codes make an allowance for the effects of relative humidity, in their deflection calculation procedures, as mentioned previously. The aim of this case study was to determine how the relative humidity would influence the long-term deflection and how the propping time would be affected. In order to do this it was decided to calculate the long term deflection values for different RH values ranging from 50% - 90% and at different ages of loading. The same concrete strength was used as in the first case study. A concrete mix with a class 42.5N cement type was used. The age of loading ranges from 3 days to 14 days, as per the previous case study. Figure 23, Figure 24 and Figure 25 show a comparison of the results obtained for each of the code based calculations. In all three cases, there is a decrease in the long-term deflection as the RH value increases. This is as expected, as the deflection due to shrinkage and creep effects reduces at increased RH levels, as was mentioned in Chapter 2. RC members that are exposed to environments with low RH values tend to experience greater drying creep and shrinkage strains. This is especially true in this case, as the age of loading coincides with the age of propping removal and the RC member is therefore not allowed to dry out before the load is applied.

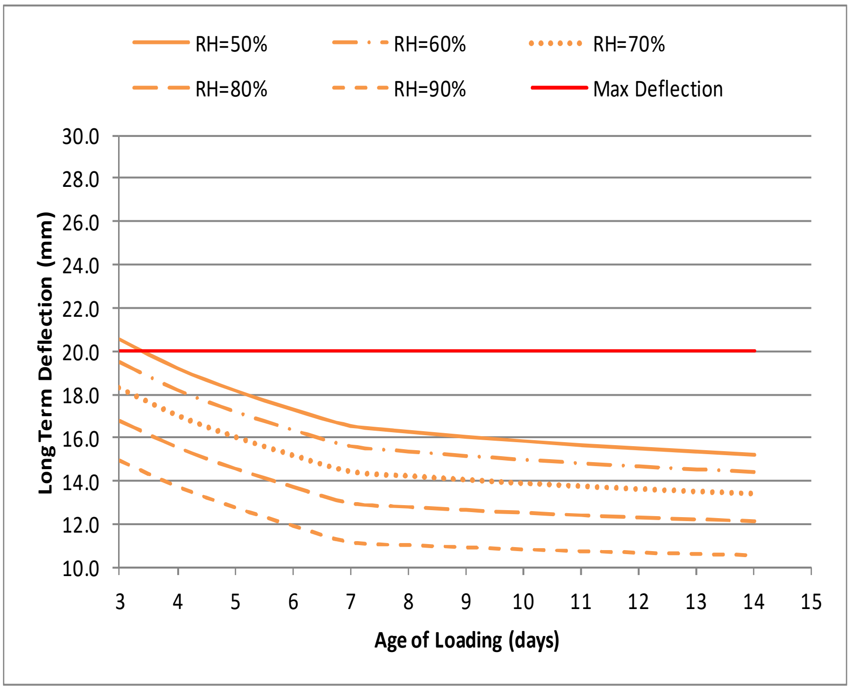

Figure 23: Effects of RH on long-term deflection based on SANS calculation procedure

The Eurocode seems to be the most sensitive to changes in relative humidity, as the deflection values between the lowest and highest humidity levels showed the biggest variation. The results for the ACI indicate that this method is the least sensitive to changes in relative humidity.

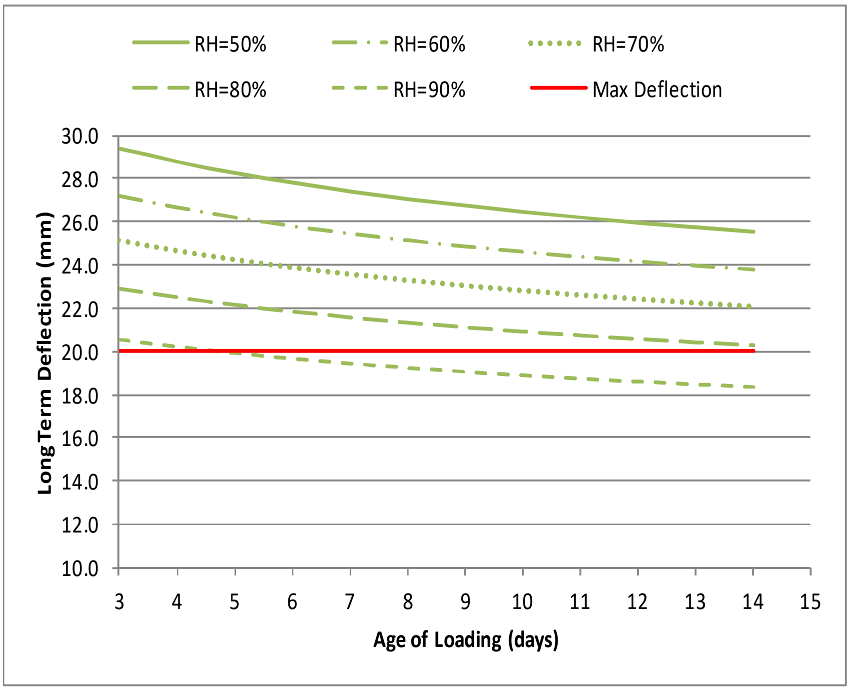

Figure 24: Effects of RH on long-term deflection based on Eurocode calculation procedure

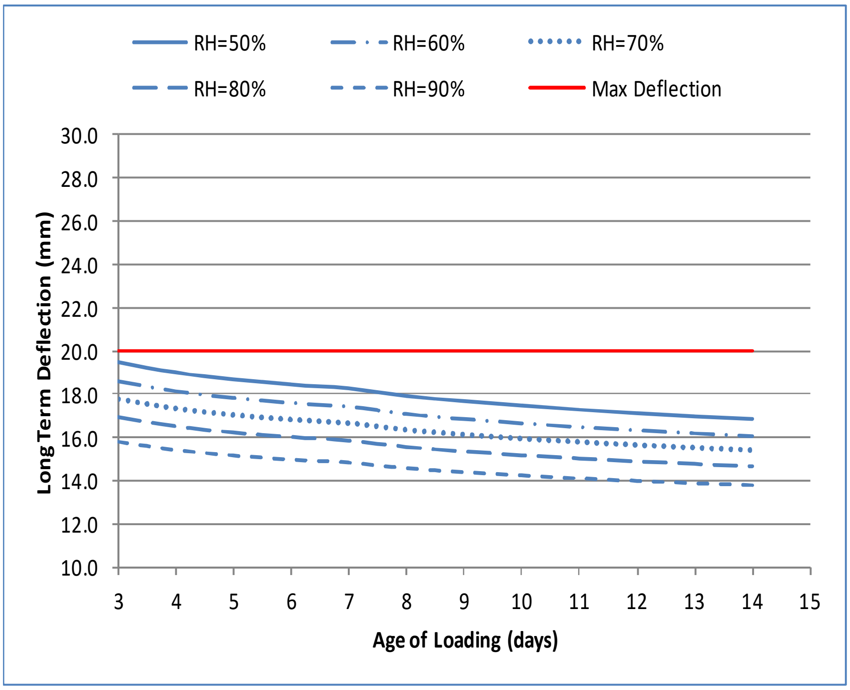

Figure 25: Effects of RH on long-term deflection based on ACI calculation procedure

This comparison indicates that at higher RH levels, the propping time may be reduced.

4.3 Modelling of Long Term Deflection for Varying Concrete Strengths

The third case study looks at the effects of concrete strength on the long term deflection after early propping removal. The aim of this investigation was to determine how the deflection would be affected for different concrete strengths and ages of loading. The SANS propping time table does not consider the effects of concrete strength. However, since the deflection calculation models are largely based on concrete strength, it is expected that there will be a noticeable change in deflection for different concrete strengths. For this case study, the relative humidity was kept at 60%, while the concrete strength ranges from 25 MPa to 60 MPa. Separate plots were generated on the same graph for different ages of loading, ranging from 3 days to 7 days. The maximum allowable deflection remains at 20 mm.

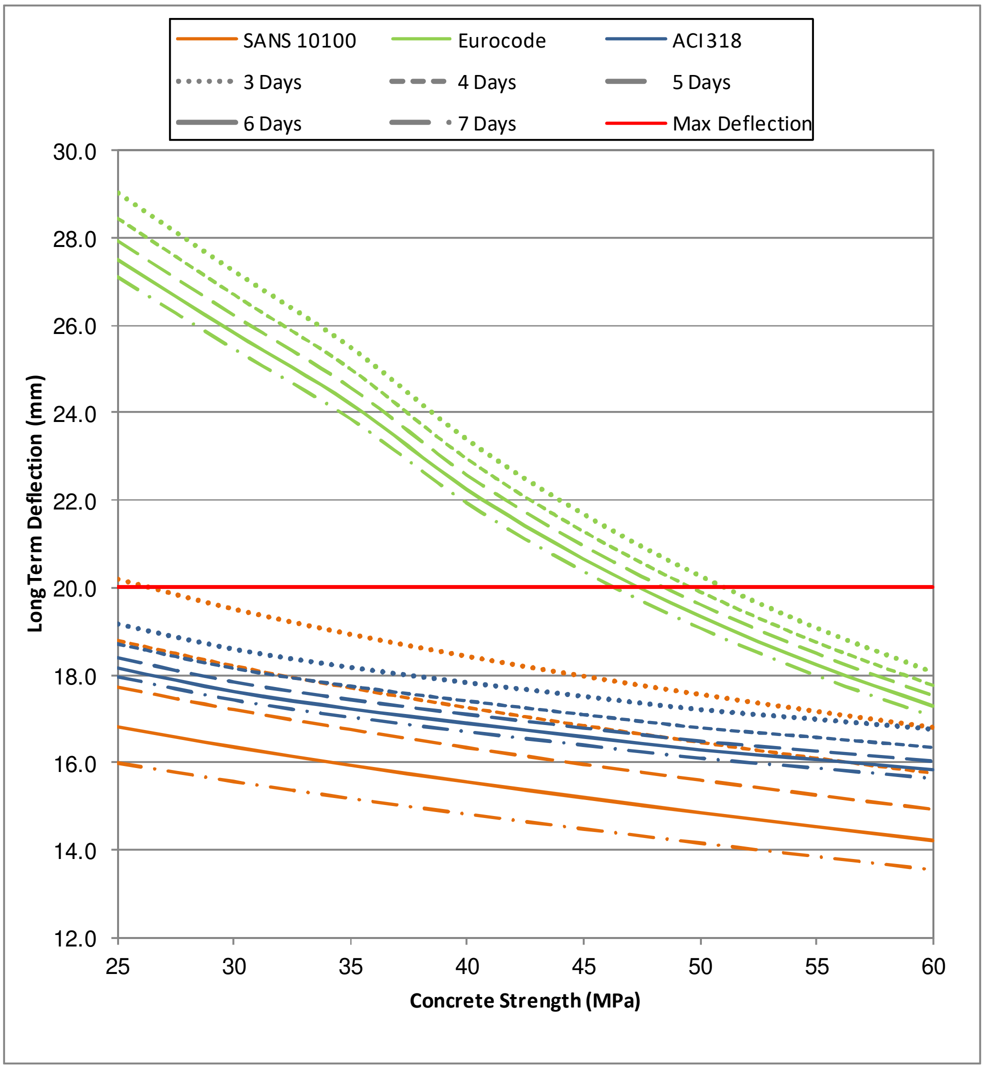

Figure 26: Effects of concrete strength on long-term deflection (age at loading 3 - 7 days)

As can be seen from Figure 26, the Eurocode code calculations seem to be significantly affected by the concrete strength, as it shows the highest reduction of the long-term deflection as the strength increases. An average reduction of 49% was recorded between the lowest and highest concrete strength values. The deflection values obtained for the ACI code only reduce by 11.5%, which is the lowest reduction of all three codes. The SANS code deflection results decrease by 15.2%. This means that the Eurocode model is more sensitive to concrete strength than the ACI and SANS code models. It is interesting to note that the ACI and SANS code calculations require a much lower concrete strength to achieve the allowable maximum deflection. The Eurocode only manages to achieve this when the concrete strength reaches approximately 45 – 50 MPa, depending on the age of loading being considered. A similar trend was observed in 4.1, where the Eurocode appears to estimate higher deflection values than the other two codes. This could again be attributed to the underestimation of the creep strains by the SANS and ACI code procedures.

Figure 27, Figure 28 and Figure 29 below show the effects of concrete strength and age of loading up to 14 days for each of the three codes individually. The relative humidity in all cases remained at 60%.

The deflection values for the SANS code indicate a gradual reduction in the deflection up to an age of 7 days. Thereafter the deflection values show only a slight reduction. In terms of propping time this indicates that there is no significant advantage gained by propping the beam beyond a period of 7 days.

The deflection values for both the ACI and Eurocode continue to reduce throughout the range of days being considered. The ACI results also show a greater initial deflection reduction between one and three days. Thereafter the deflection decreases at a reducing rate. In all three cases the deflection reduces as the concrete strength increases.

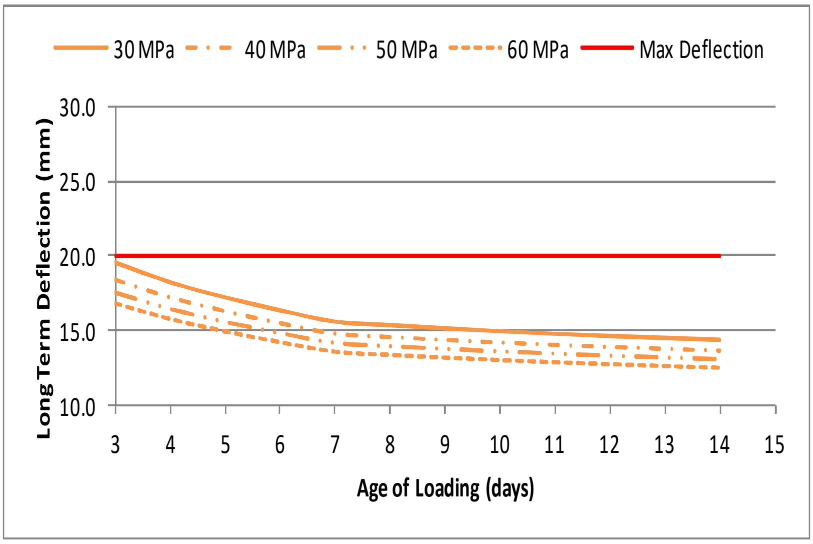

Figure 27: Effects of concrete strength and age of loading on long term deflection according to SANS 10100

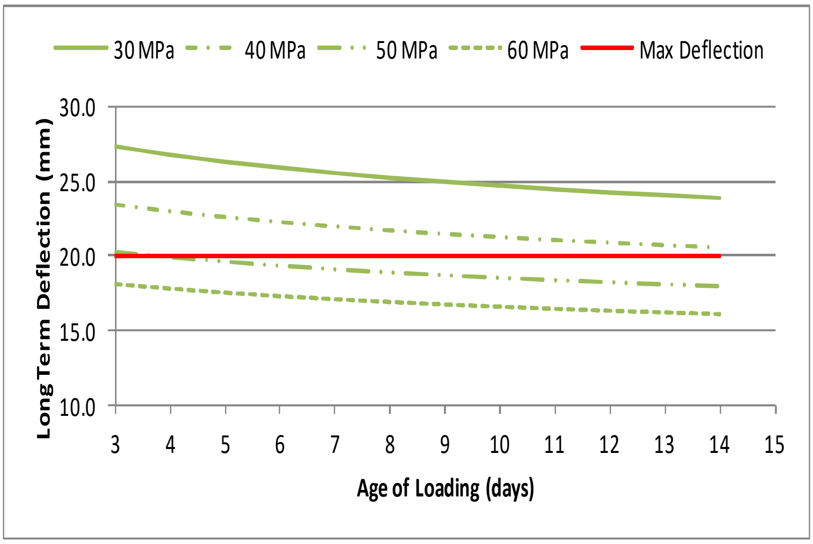

Figure 28: Effects of concrete strength and age of loading on long term deflection according to Eurocode

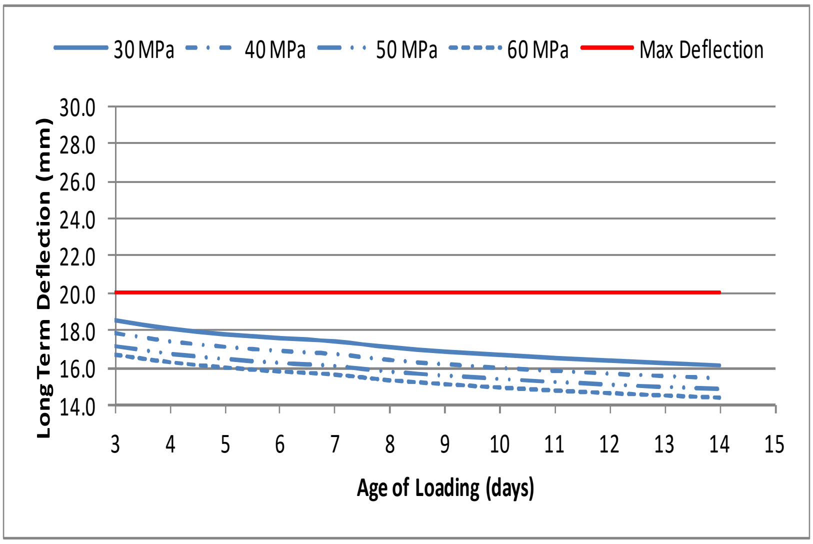

Figure 29: Effects of concrete strength and age of loading on long term deflection according to ACI 318

The results for all three codes show a reduction in deflection as the concrete strength increases. A reduction in propping time is therefore possible if the concrete strength is increased. However, an increase in strength is usually linked to an increase in the cost of the concrete. The advantage gained from the reduction in the propping time would need to get offset by the increased material costs. A cost analysis would need to be performed to determine the viability.

4.4 Results and Discussion

Based on the results obtained in the first case study, it seems possible to reduce the propping time for beams, when compared to the propping time table given in SANS 2001. With the exception of the Eurocode, the long term deflection values obtained indicate that the deflection limits are already within the allowable tolerance after approximately 3 days. However, research indicates that the SANS and ACI methods seem to underestimate the creep effects [29]. In addition to this, the material properties used in the calculations for all three codes are mostly based on the concrete compressive strength. As was discussed in Chapters 3, the relationship between the concrete compressive strength and elastic modulus is not straight forward. The accuracy of these results is therefore questionable. Also, there are numerous factors which affect the concrete compressive strength development, most of which are not being considered by these code based models. One of these factors is the cement type, for which only the Eurocode and ACI make an allowance for in their respective concrete strength calculation procedures. Structural designers using either one of these codes therefore have the advantage of calculating the estimated long term deflection more accurately or at the very least be able to determine what the effects might be if the cement type is changed. The SANS code does not make this distinction. Interestingly, the propping time table provides different propping times based on the cement type used in the mix design, as was shown in Chapter 1. Further research needs to be done to determine exactly how significant the underestimation of the creep effects are for the SANS and ACI codes and how these affect the accuracy of the calculated deflection results.

The results from the second case study show that the relative humidity can influence the long term deflection, especially at early ages of loading. This indicates that the propping time is also affected by the relative humidity. Designers will usually be able to obtain values for the ambient relative humidity quite easily if they know the geographical location of the structure. The propping time table does not consider the effects of relative humidity. This could be a worthwhile addition to the table to allow for more accurate propping time estimates.

According to the results obtained in the third case study, the concrete strength can also affect the long term deflection. These results are not surprising, as the code-based models use the concrete compressive strength to derive material properties, such as the elastic modulus. The concrete propping times provided by the SANS code do not distinguish between different concrete strengths.

In summary it can be said that the propping time table in the SANS code should be reviewed and amended where possible. An allowance should be made for the relative humidity and concrete strengths. This will allow structural designers to assess the minimum propping times more accurately.

5. Calculation Tool for Propping Time

This chapter gives a brief introduction and description of a propping time calculation tool, which was developed in MS Excel and is linked to the deflection modelling spreadsheet that was used in Chapter 4. Figure 30 below shows the interface of the deflection calculator. The highlighted cells contain parameters which can be edited by the user as needed. The section size is as provided in the Input tab of the MS Excel Spreadsheet.

Case Study 4: Calculation of minimum propping time

Objective:	To determine the minimum propping time for a range of deflection values, using a given member size, concrete strength, area of steel and relative humidity
Design input:	The beam geometry, concrete strength, area of steel and RH remains constant, while the deflection varies as specified

Concrete (Mpa):	**35**
Area of Steel (mm²):	**1611**
Relative humidity (%):	**60**
Max Deflection (mm):	**24**
Min Deflection (mm):	**15**
Step size (mm):	**1**

	Propping Time (days)		
Deflection (mm)	**SANS**	**Eurocode**	**ACI**
15.0	7.8	556.9	23.6
16.0	5.9	300.7	12.0
17.0	4.7	171.3	7.0
18.0	3.7	101.8	3.4
19.0	2.9	62.4	1.9
20.0	2.3	39.3	1.2
21.0	1.8	25.1	0.8
22.0	1.4	16.2	0.6
23.0	1.1	10.4	0.4
24.0	0.9	6.6	0.3

Propping time Reset

Propping time SANS

Propping time Eurocode

Propping time ACI

Figure 30: Screen grab of interface for propping time calculation tool

Based on a given concrete strength, area of steel, RH and a deflection range, the minimum propping time is calculated using the code-based methods, as discussed in Chapter 3. This is done by using the goal seek function in Excel, which is implemented in a VBA code and assigned to control buttons to allow the user to calculate the propping times for different input conditions. The propping time can easily be manipulated by adjusting the concrete strength, area of steel or section size. Figure 31 and Figure 32 are examples of the calculated propping times for the three codes being used in this research. The section size is the same as used in the previous chapter and the concrete strength, area of steel and RH-value is as shown in Figure 30.

Predictably, the propping durations for the SANS and ACI codes are much shorter than for the Eurocode. This is not surprising, as the Eurocode also achieved the highest deflection values at early propping removal times, as was discussed in Section 4.1.

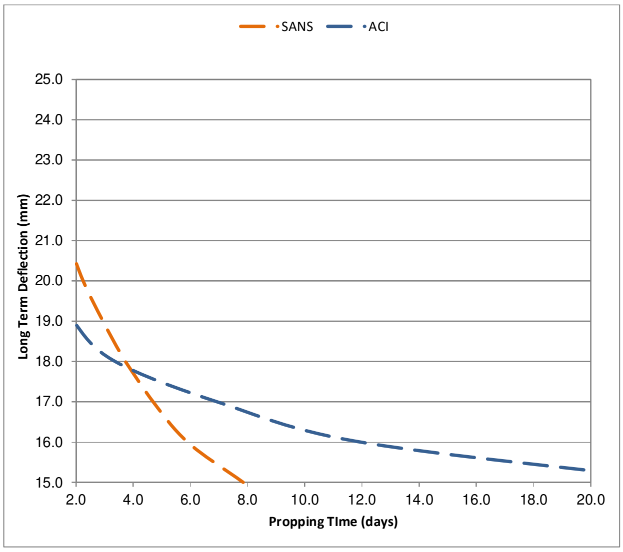

Figure 31: Calculated propping times based on SANS and ACI methods

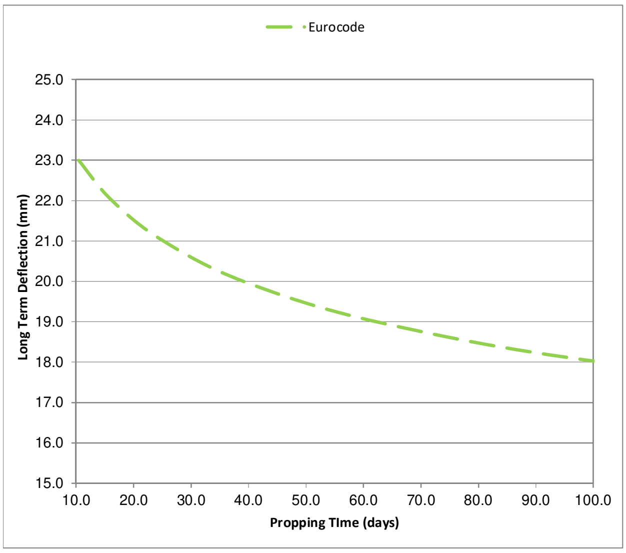

Figure 32: Calculated propping times based on Eurocode method

This propping time calculation tool could be used to optimise the section size, concrete strength and area of steel in order to achieve a desired propping time. However, since the code-based deflection calculation procedures only give a rough indication of the expected deflection values, the propping times obtained by this calculation tool could at best be used as a rough guide for now. This propping time tool can easily be developed further. For example, if more accurate shrinkage or creep models are incorporated into any of the design codes used in this research, then these can easily be programmed into the spreadsheet to give more accurate results.

6. Conclusion and Recommendations

6.1 Conclusions

The results obtained in Chapter 4 seem to indicate that the propping times given in the SANS 2001 design code might be unduly conservative. Both the SANS and ACI code-based deflection results seem to indicate that a considerable reduction in propping time might be possible without exceeding the maximum allowable deflection limitations. However, the results obtained from the Eurocode contradict these findings. Also, both the SANS and ACI codes have got some limitations, which could affect the accuracy of the results obtained. The SANS code does not provide a method for calculating the concrete compressive strength for different ages, which is one of the core parameters of all three code-based deflection calculation methods. It also does not differentiate between different cement types, which can influence the rate at which the concrete strength develops. The ACI code on the other hand prescribes a minimum curing period of seven days when calculating the long-term effects. It therefore seems to be less suitable for calculating the estimated long-term deflection for curing periods that are shorter than 7 days. In addition to this, some research indicates that both the ACI and SANS codes seem to under predict the long-term creep effects. In terms of propping time, this means that the actual propping period might be longer than the calculated amount.

The results in Chapter 4 also show that the concrete strength and RH value have got a noticeably affect on deflection. According to the results obtained from the code-based deflection calculation procedures, a higher RH value or concrete strength causes a reduction in the deflection and could therefore result in a reduced propping time. Both of these parameters are not explicitly taken into consideration by the propping time table in SANS 2001.

In summary it can be said that since the code-based deflection calculation procedures are only suitable to give a rough indication of the short and long-term deflections, the same should also be applicable to the propping times obtained in this research. An improvement in the accuracy of the deflection prediction models would also improve the accuracy of the propping duration prediction. All of the findings from this research were purely based on the code-based deflection models. Also, the research was limited to a continuous beam configuration. In order to be able to draw some general practical conclusions one would have to investigate further case studies for different beam configurations and conduct some practical experiments to be able to verify the results.

The propping time calculator discussed in Chapter 5 could be a useful tool for structural designers to determine the shortest possible propping durations. For now, this tool should only be used to give a rough estimate of the propping duration, as the accuracy of the calculated propping time is linked to the accuracy of the estimated deflection values. However, if the accuracy of the deflection models can be improved, then the propping duration calculator could be used to give more accurate results.

6.2 Recommendations

Based on the findings of this research, the following recommendations can be made:

- The results obtained in this research need to be verified using practical experiments.
- The effects of early propping removal also need to be investigated on other beam configurations, i.e. simply supported, cantilever, in order to generalise the conclusions
- Additional modelling for different load cases, member sizes and area of steel needs to be done to determine their effects on deflection and propping times
- The suitability of using the deflection calculation methods to determine the propping duration for RC slabs needs to be further investigated
- The propping time table given in the SANS document should be reviewed and improved to include the effects of relative humidity and concrete strength
- A method for calculating the concrete strength and elastic modulus for different concrete ages needs to be developed for the SANS code
- The deflection calculation methods need to be reviewed to determine how their accuracy can be improved with the aim of allowing more accurate propping time predictions
- Structural designers usually only specify a minimum 28 day concrete strength. The plausibility of specifying additional parameters, such as minimum rate of strength development, or E-modulus development in order to be able to predict propping times more accurately needs to be investigated.

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Appendix A – Deflection limits according to SANS 10160-1

The following tables were reproduced from SANS 10160-1.

Table 12: Summary of recommended criteria for the irreversible serviceability limit state

1	2	3	4	5	6	7	8	9	10	11	12
Deformation	Effect	Criterion	Actions and deflections [a]							Clause	Conditions and comments
			Constrution deviation and camber	Differential setlement	Structural self-weight	Non-structural self-weight	Pre-stressing	Imposed load	Wind		
Medial deflection of floors	Damage at supports	Span/300			C^b	E^c C_b	C^b	E^c C_b		C.7.2	Also D.6.2
	Ceiling damage	-			C^b	E^c C_b	C^b	E^c C_b		C.7.3	Varies with construction Usually less critical than partitions
	Partition damage Partition isolated from floor (for span/height <3,5)	Span/500 to Span/300									Partition follows movement of floor beneath
		10 mm			C^b	E^c C^b	C^b	E^c C_b		C.7.4	Excessive deflection of floor below partition
		10 mm to 15 mm									Excessive deflection of floor or roof above partition
Medial deflection of roofs or roof members	Damage at supports	Span/300			C^b	E^c C_b	C^b	E^c C_b		C.6.2.3	
	Ceiling damage	-			C^b	E^c C_b	C^b	E^c C_b		C.7.3	Varies with construction
	Partition damage - isolated (for span/height <3.5)	10 mm to 15 mm			C^b	E^c C_b	C^b	E^c C_b		C.7.4	Deflection at nodes for truss
	Roof covering damage	Span/250 to Span/125			C^b	E^c C_b	C^b	E^c C_b	E^c	C.7.5	

[a] Columns 4 to 10 indicate which actions and displacements are to be considered when calculating compliance of the structure with the given criterion.

[b] Creep effect.

[c] Elastic effect.

Table 13: Summary of recommended criteria for the irreversible serviceability limit state (concluded)

1	2	3	4	5	6	7	8	9	10	11	12
Deformation	Effect	Criterion	Actions and deflections [(a)]							Clause	Conditions and comments
			Constrution deviation and camber	Differential setlement	Structural self-weight	Non-structural self-weight	Pre-stressing	Imposed load	Wind		
Terminal deflection of cantilever floors	Ceiling Damage	-			C^b	E^c C_b	C^b	E^c C_b		C.7.3	Varies with construction
	Partition damage	Span/500 to Span/300			C^b	E^c C_b	C^b	E^c C_b		C.7.4	
Terminal deflection of cantilever roofs	Ceiling damage	-			C^b	E^c C_b	C^b	E^c C_b		C.7.3	Varies with construction
	Partition damage - Isolated	10 mm to 15 mm			C^b	E^c C_b	C^b	E^c C_b		C.7.4	Deflections at nodes for truss
	Roof covering damage	Span/250 to Span/125			C^b	E^c C_b	C^b	E^c C_b	E^c	C.7.5	
Terminal deflection of non-cantilever horizontal members	Damage at supports	Span/100		E^c C_b						C.6.2.3	
	Partition damage	Span/500		E^c C_b						C.7.4	
Terminal deflection of vertical members	Damage at supports	Story height/100							E^c	C.6.2.3	
	Partition damage	Storey height/500							E^c	C.7.4	Partition damage

[a] Columns 4 to 10 indicate which actions and displacements are to be considered when calculating compliance of the structure with the given criterion.

[b] Creep effect.

[c] Elastic effect.

Table 14: Summary of recommended criteria for the reversible and long-term serviceability limit state

1	2	3	4	5	6	7	8	9	10	11	12
			Actions and deflections [(a)]								
Deformation	Effect	Criterion	Constrution deviation and camber	Differential setlement	Structural self-weight	Non-structural self-weight	Pre-stressing	Imposed load	Wind	Clause	Conditions and comments
Medial deviation of floors	Appearance	Visible length/250 or 30 mm	Yes		C^b	E^c C_b	C^b	E^c C_b		C.8.1	
	Use (curvature)	Span/300	Yes		C^b	E^c C_b	C^b	E^c C_b		C.9.1	
Medial deviation of roofs or roof members	Appearance	Visible length/250 or 30 mm	Yes		C^b	E^c C^b	C^b	E^c C_b		C.8.1	Quasi permanent component of imposed load
Terminal deviation of cantilever floors	Appearance	Visible length/250 or 15 mm	Yes	E^b C^c	E^b C^c	E^b C^c		E^b C^c		C.8.1	
	Use (curvature)	Span/125	Yes	E^b C^c	E^b C^c	E^b C^c		E^b C^c		C.9.1	
	Use (rotation)	Span/100	Yes	E^b C^c	C^b	E^c C_b	C^b	E^c C_b		C.9.2	
Terminal deviation of cantilever roofs	Appearance	Visible length/250 or 15 mm	Yes	E^b C^c	E^b C^c	E^b C^c		E^b C^c		C.8.1	Quasi permanent component of imposed load

[a] Columns 4 to 10 indicate which actions and displacements are to be considered when calculating compliance of the structure with the given criterion.
[b] Elastic effect.
[c] Creep effect.

Table 15: Summary of recommended criteria for the reversible and long-term serviceability limit state (concluded)

1	2	3	4	5	6	7	8	9	10	11	12
Deformation	Effect	Criterion	Actions and deflections [(a)]							Clause	Conditions and comments
			Constrution deviation and camber	Differential setlement	Structural self-weight	Non-structural self-weight	Pre-stressing	Imposed load	Wind		
Terminal deviation of non-cantilever horizontal members	Use (slope)	Span/100	Yes	E[b] C[c]						C.9.1	
Terminal deviation of vertical members	Appearance	Storey height/250	Yes	E[b] C[c]	E[b] C[c]	E[b] C[c]		E[b] C[c]		C.8.2	Where self-weight, imposed and wind loads act eccentrically
Oscillations of members	Resonance	-						E[b]	E[b]	C.9.3	
	Use	-						E[b]	E[b]		
Oscillations of buildings as a whole	Use	-						E[b]	E[b]		
Horizontal terminal deflection of high-rise buildings		Building height/500									

[a] Columns 4 to 10 indicate which actions and displacements are to be considered when calculating compliance of the structure with the given criterion.

[b] Elastic effect.

[c] Creep effect.

Appendix B – WITS Model Coefficients

The following table contains the covariates used in the WITS model.

Stone Type	Cement Type										
	CEM I	CEM II A-D	CEM II A-L	CEM II A-M(L)	CEM II A-S	CEM II A-V	CEM II B-M (V/L)	CEM II B-S	CEM II B-V	CEM III A	CEM V A
Andesite	X		X	X	X		X	X	X	X	X
Dolerite	X	X	X		X	X		X	X	X	
Dolomite	X							X		X	
Granite	X							X	X	X	
Greywacke	X					X			X	X	
Pretoria Quartzite	X										
Quartzite	X										
Shale										X	
Tillite	X							X	X	X	
Wits Quartzite	X									X	

Sand Type	Cement Type										
	CEM I	CEM II A-D	CEM II A-L	CEM II A-M(L)	CEM II A-S	CEM II A-V	CEM II B-M (V/L)	CEM II B-S	CEM II B-V	CEM III A	CEM V A
Andesite	X										
Cape Flats	X					X			X		
Dolerite	X	X							X	X	
Dolomite	X		X		X	X		X	X	X	
Ecca Grit									X		
Granite	X		X	X			X	X	X	X	X
Klipheuwel Pit	X									X	
Natural	X				X			X		X	
Pretoria Quartzite	X										

Quartzite (up to 80%*)	X										
River (up to 25%*)	X									X	
River Vaal (up to 20%*)	X							X	X	X	
Shale										X	
Tillite (up to 80%*)	X							X	X	X	
Wits Quartzite	X									X	

*Indicates maximum proportion of sand type in total sand content

Sand Type	**Stone Type**									
	Andesite	Dolerite	Dolomite	Granite	Greywacke	Pretoria Quartzite	Quartzite	Shale	Tillite	Wits Quartzite
Andesite	X									
Cape Flats					X					
Dolerite		X								
Dolomite		X	X							
Ecca Grit		X								
Granite	X			X						
Klipheuwel Pit					X					
Natural	X	X								
Pretoria Quartzite						X				
Quartzite							X			
River										X
River Vaal	X		X	X	X	X	X		X	X
Shale										X
Tillite									X	
Wits Quartzite								X		X

Appendix C – List of Deflection Calculation Expressions for SANS, ACI & Eurocode

Deflection calculation formulas according to design codes

Calculation Step	SANS	ACI	Eurocode	Calculation Step
Immediate deflection	$\Delta_i = KM_s \frac{l^2}{E_c I_e}$	$\Delta = KL^2 \frac{M}{E_c I_e} = \mathrm{K}\mathrm{L}^2 \frac{1}{\mathrm{r}}$	$\Delta = KL^2 \frac{M}{E_c I_e} = KL^2 \frac{1}{r_n}$ $\frac{1}{r_n} = \zeta \frac{M_{QP}}{E_{eff} I_c} + (1-\zeta) \frac{M_{QP}}{E_{eff} I_u}$ Degree of cracking: $\zeta = 1 - 0.5\left(M_{cr}/M_{QP}\right)^2$ $\zeta = 1 - \left(M_{cr}/M_{QP}\right)^2$	Immediate deflection Flexural curvature Long term deflection Immediate deflection
Cracked moment of inertia	$I_{cr} = \frac{bx^3}{12} + bx\left(\frac{x}{2}\right)^2 + A_s(d-x)^2$ $x = \alpha_e \rho \left[\sqrt{1 + \frac{2}{\alpha_e \rho}} - 1\right] d$ $\alpha_e = \frac{E_s}{E_c}$ $\rho = \frac{A_s}{bd}$	With compression reinforcement: $I_{cr} = \frac{bx^3}{3} + \alpha_e A_s (d-x)^2 + (\alpha_e - 1)A'_s(x-d')^2$ $x = [\{2dB(1 + rd'/d) + (1+r)^2\}^{0.5} - (1+r)]/B$ Without compression reinforcement: $I_{cr} = \frac{ba^3}{3} + \alpha_e A_s (d-x)^2$ $x = [\{2dB + 1\}^{0.5} - 1]/B$ Where $B = b/(\alpha_e A_s)$ $r = (\alpha_e - 1)A'_s/\alpha_e A_s$	$I_c = \frac{bx_c{}^3}{3} + \alpha_e A_s (d - x_c)^2 + (\alpha_e - 1)A_{s2}(x_c - d_2)^2$ $x_c = \left\{ \begin{array}{c} \left[\left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right)^2 + 2b\left(A_s d\alpha_e + A_{s2} d_2 (\alpha_e - 1)\right)\right]^{0.5} \\ -\left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right) \end{array} \right\} \Big/ b$ Effective modulus ratio: $\alpha_e = {E_s}\big/{E_{eff}}$ Effective modulus: $E_{c.eff} = \frac{E_{c28}}{1+\varphi(\infty, t_0)}$	Cracked moment of inertia Effective modulus for long term deflection
Cracking moment	$M_{cr} = \frac{f_r I_g}{y_t}$	$M_{cr} = \frac{f_r I_g}{y_t}$	$M_{cr} = \frac{0.9 f_{ctm} I_u}{h - x_u}$	Cracking moment
Uncracked moment of inertia	$I_u = \frac{bh^3}{12}$ $x_g = \frac{h}{2}$	$I_u = \frac{bh^3}{12}$ $x_g = \frac{h}{2}$	$I_u = \frac{bh^3}{12} + bh\left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1)[A_s(d - x_u)^2 + A_{s2}(x_u - d_2)^2]$ $x_u = \frac{\frac{bh^2}{2} + (\alpha_e - 1)(A_s d + A_{s2} d_2)}{bh + (\alpha_e - 1)(A_s + A_{s2})}$	Uncracked moment of inertia
Concrete Elastic Modulus	$E_{c,28} = K_0 + \alpha f_{cu,28}$ Refer to table 8.2 in Fultons for K_0 and α values. Min. Concrete strength: 20MPa $f_{cu,28}$ = cube strength at 28 days $E_{c,t} = K_0 + \alpha f_{cu,t}$ $f_{cu} \geq 20MPa$ $E_{c,t} = E_{c,28}(0{,}4 + 0{,}6 f_{cu,t}/f_{cu,28})$ $E_{c,28} = K_0 + 0{,}2 f_{cu,28}$	$E_c = w_c{}^{1.5} \cdot 0.043\sqrt{f_c'}$ for concrete weight $w_c = 1440 - 2480 kg/m^3$; f_c' = cylinder strength or $E_c = 4700\sqrt{f_c'}$ for normal weight concrete ($w_c = 2300 kg/m^3$)	$E_{cm} = 22[(f_{ck} + 8)/10]^{0.3}$ at 28 days where f_{ck}=characteristic cylinder strength. Allowance for aggregate type: reduce by 10% for limestone and 30% for sandstone, increase by 20% for basalt For ages other than 28 days: $E_{cm(t)} = \left(f_{cm(t)}/f_{cm}\right)^{0.3} E_{cm}$ $f_{cm} = f_{ck} + 8$; $f_{cm(t)} = \beta_{cc(t)} f_{cm}$ and $\beta_{cc(t)} = exp\left\{s\left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\}$ with cement type coefficient s = 0.2 (CEM42.5R, CEM52.5N & CEM52.5R, Class R); s = 0.25 (CEM32.5R, CEM42.5N, Class N); s = 0.38 (CEM32.5N, Class S)	

Modulus of rupture (unrestrained)	$f_r = 0{,}65\sqrt{f_{cu}}$	$f_r = 0{,}623\sqrt{f_c'}$ Compressive strength at any time t $f_{cmt} = \left[\frac{t}{a+bt}\right] f_{cm28}$ a and b are constants which are a function of the cement type and the type of curing used	$f_{ctm} = \left(0.3 f_{cm}^{(2/3)}\right) \leq 60MPa = f_{cu}$ $or\ 2.12\ln(1 + (f_{ck} + 8)/10) > 60MPa = f_{cu}$ Tensile strength at time t: $f_{ctm(t)} = \left(\beta_{cc(t)}\right)^{\alpha} f_{ctm}$ where $\alpha = 1$ for $t < 28days$ or $\alpha = 2/3$ for $t \geq 28days$	Concrete tensile strength
Modulus of rupture (restrained)	$f_r = 0{,}30\sqrt{f_{cu}}$		$f_{ctm} = \left(0.3 f_{cm}^{(2/3)}\right) \leq 60MPa$ $or\ 1.08\ln(f_{cm}) + 0.1 > 60MPa$	Concrete tensile strength (striking at < 7d, or constr. overload is taken into account)
Effective moment of inertia	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$	$I_e = \frac{I_{cr}}{1 - \left(1 - \frac{I_{cr}}{I_u}\right)\left(\frac{M_{cr}}{M_a}\right)^2} \leq I_u$	Effective moment of inertia
Long term creep deflection	$\Delta_{\infty} = \lambda\Delta_i = (1 + x_i\Phi)\Delta_i$ $x_i = \frac{x}{d} = \alpha_e \rho\left[\sqrt{1 + \frac{2}{\alpha_e \rho}} - 1\right]$	$\Delta_l = k_r \cdot \phi \cdot \Delta_i$ Reduction factor: $k_r = \frac{0.85}{(1+50 * \rho')}$ $\rho' = A_s'/bd$	$\delta_{QP} = KL^2 \frac{1}{r_{t,QP}}$ $\frac{1}{r_{t,QP}} = \frac{1}{r_n} + \frac{1}{r_{cs}}$	Long term deflection Total curvature
Effective thickness Shrinkage deflection	$\frac{2A}{Exposed\ perimeter}$ $\Delta_{cs} = K_{sh} k_{cs} \frac{\varepsilon_s L^2}{h}$	$\Delta_{cs} = K_{sh} L^2 \frac{1}{r_{cs}}$	$h_0 = \frac{2A_c}{u}$	Notional member size
Shrinkage curvature		$\frac{1}{r_{cs}} = 0.7 \frac{\varepsilon_{sh}}{h} (\rho - \rho')^{1/3} \left[\frac{\rho - \rho'}{\rho}\right]^2$ $\rho = 100A_s/bd$ $\rho' = 100A_s'/bd$ $\rho - \rho' \leq 3.0\%$	$\frac{1}{r_{cs}} = \zeta\varepsilon_{cs}\alpha_e \frac{S_c}{I_c} + (1 - \zeta)\varepsilon_{cs}\alpha_e \frac{S_u}{I_u}$ $S_u = A_s(d - x_u) - A_{s2}(x_u - d_2)$ $S_c = A_s(d - x_c) - A_{s2}(x_c - d_2)$	Shrinkage curvature
Shrinkage strain		$\varepsilon_{sh}(t, t_c) = \frac{(t - t_c)^{\alpha}}{f + (t - t_c)^{\alpha}} \times \varepsilon_{shu}$ $\alpha = 1$ and $f = 26.0e^{\{1.42 \times 10^{-2}(V/S)\}}$	$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$	Shrinkage strain
			$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}$ $\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}}$	Drying shrinkage strain
			$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty)$ $\beta_{as}(t) = 1 - exp(-0.2t^{0.5})$ $\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10)10^{-6}$	Autogenous shrinkage
Ultimate shrinkage strain		$\varepsilon_{shu} = 780 \times 10^{-6}\gamma_{sh}$	$\varepsilon_{cd,0} = 0.85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH}$ $\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right]$; α_{ds1} and α_{ds2} factors for cement type	Basic drying shrinkage

Reduction factor		$\gamma_{sh} = \gamma_{sh,tc}\gamma_{sh,RH}\gamma_{sh,vs}\gamma_{sh,s}\gamma_{sh,\Psi}\gamma_{sh,c}\gamma_{sh,\alpha}$	$\varphi_{RH} = 1 + \frac{1-RH/100}{0.1\sqrt[3]{h_0}}$ for $f_{cm} \leq 35MPa$ $\varphi_{RH} = \left[1 + \frac{1-RH/100}{0.1\sqrt[3]{h_0}} \cdot \alpha_1\right]\alpha_2$ for $f_{cm} \geq 35MPa$	Relative humidity effects
Initial moist curing coefficient		$\gamma_{sh,tc} = 1.202 - 0.2337\log(\mathrm{t_c})$	$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$	Concrete strength effects
Relative humidity coefficient		$\gamma_{sh,RH} = \begin{Bmatrix} 1.40 - 1.02h\ for\ 0.40 \leq h \leq 0.80 \\ 3.00 - 3.0h\ for\ 0.80 \leq h \leq 1 \end{Bmatrix}$	$\beta(t_0) = \frac{1}{(0.1 + {t_0}^{0.20})}$	Concrete age at loading effects
Volume to surface area coefficient		$\gamma_{sh,vs} = 1.2e^{\{-0.00472\,(V/S)\}}$		
Slump coefficient		$\gamma_{sh,s} = 0.89 + 0.00161s$		
Fine aggregate content coefficient		$\gamma_{sh,\Psi} = \begin{Bmatrix} 0.30 + 0.014\Psi\ for\ \Psi \leq 50\% \\ 0.90 + 0.002\Psi\ for\ \Psi > 50\% \end{Bmatrix}$		
Cement content coefficient		$\gamma_{sh,c} = 0.75 + 0.00061c$		
Air content coefficient		$\gamma_{sh,\alpha} = 0.95 + 0.008\alpha \geq 1$		
Creep coefficient		$\phi(t,t_0) = \frac{(t-t_0)^{\Psi}}{d + (t-t_0)^{\Psi}}\phi_u$ where $\Psi = 1.0$ and $d = f$ if shape and size effects are considered $(d = f = 26.0e^{[1.42\times10^{-2}(V/S)]})$	$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0)$	Creep coefficient
			$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$	Notional creep coefficient
Correction coefficient		$\gamma_c = \gamma_{c,t0}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\Psi}\gamma_{sh,\alpha}$	$\beta_c(t,t_0) = \left[\frac{(t-t_0)}{(\beta_H + t - t_0)}\right]^{0.3}$	Development of creep with time after loading
Ultimate creep coefficient		$\phi_u = 2.35\gamma_c$	$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500$ for $f_{cm} \leq 35$ $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3$ for $f_{cm} \geq 35$	Relative humidity factor
Moist curing coefficient		$\gamma_{c,t0} = 1.25{t_0}^{-0.118}$ (age at application of load greater than 7 days)	$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7}$ $\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2}$ $\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5}$	Influence of concrete strength
Steam curing coefficient		$\gamma_{c,t0} = 1.13{t_0}^{-0.094}$ (age at application of load greater than 1-3 days)	$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + {t_{0,T}}^{1.2}} + 1\right)^{\alpha} \geq 0.5$ where α = -1 for Class S, 0 for Class N and 1 for Class R cement	Modified time of loading: Effects of cement type
Relative humidity coefficient		$\gamma_{c,RH} = 1.27 - 0.67h$ for RH $h \geq 0.40$	$t_T = t_{0,T} = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i$	Temperature adjusted concrete age
Volume to surface area coefficient		$\gamma_{c,vs} = \frac{2}{3}\left(1 + 1.13e^{\{-0.0213\,(V/S)\}}\right)$		

Slump coefficient Fine aggregate content coefficient		$\gamma_{c,s} = 0.82 + 0.00264s$ $\gamma_{c,\Psi} = 0.88 + 0.0024\Psi$		
Air content coefficient Simplified long term deflection		$\gamma_{c,\alpha} = 0.46 + 0.09\alpha \geq 1$ $\Delta_t = \lambda_\Delta \times \Delta_i$ $\lambda_\Delta = \dfrac{\xi}{1 + 50\rho'}$	$f_{ck}(t) = f_{cm}(t) - 8$ for $(3 < t < 28\ days)$	Characteristic cylinder strength at time t

Appendix D – Screen Shots of Excel Model

Input Data Sheet

Beam Geometry and Section Properties

Beam type/support condition	Both ends continuous		Number of spans:	4
Effective span of member:	L =	5000	mm	
Section width:	b =	280	mm	
Overall section depth:	h =	450	mm	
Concrete cover depth	c =	30	mm	
Shear reinforcement diamete	$\varnothing_{Asv}$ =	10	mm	
Tension rebar diameter	$\varnothing_{As}$ =	20	mm	
Compression rebar diameter	$\varnothing_{Asc}$ =	12	mm	
Depth to tension rebar	d =	400	mm	
Depth to compression rebar	d_2 =	46	mm	
Moment of inertia of section	I_g =	2.13E+09	mm^4	
Deflection coefficient	K =	0.080		
Area of tension steel	A_s =	1611	mm²	Area of tension steel over supports: 1787 mm² (Only applicable for continuous beams)
Area of compression steel	A_{s2} =	402	mm²	
Beam restraint type	Unrestrained			
Max allowable deflection	Δ_{max}	20	mm	

Material Properties

Reinforcement yield strength	f_y =	450	MPa
Concrete strength (cube strength)	f_{cu} =	30	MPa
Reinforcement elastic modulus	E_s =	200	GPa
Aggregate type	Greywacke		
Cement type	CEM42.5N		
Cement content	c =	160	kg/m³
Air content	α =	N/A	%
Slump	s =	75	mm
Fine aggregate content	Ψ =	46.6	%
Age of concrete at loading	t_o =	3	days
Curing type	Moist-cured		
Curing duration	t_c =	3	days
Concrete density (kN/m³)	γ_c =	24	kN/m³
Age of concrete for Creep calculation	t=	10950	days
Age of concrete for Shrinkage calc.	t=	10950	days
Relative humidity	RH =	60	%
		60	

Loads and moments

Permament load on beam (UDL)	G_k =	30	kN/m
Imposed load on beam (UDL)	Q_k =	25	kN/m
Percentage of permanent imposed load	% =	80	
Max moment at midspan (SLS)	M_{QP} =	108.4	kNm
Max moment over support No. 2 (SLS)	M_B =	-150.67	kNm
Max moment over support No. 3 (SLS)	M_C =	-99.979	kNm

SANS10100-1: DEFLECTION CALCULATION

Immediate deflection:

Beam Geometry and Section Properties

Effective span of member:	l =	5000	mm
Section width:	b =	280	mm
Section depth:	d =	450	mm
Concrete cover depth	c =	30	mm
Shear reinforcement diameter	$\varnothing_{Asv}$ =	10	mm
Reinforcement bar diameter	$\varnothing_{As}$ =	20	mm
Effective section depth:	d_{ef} =	400	mm
Moment of inertia of section:	I_g =	2.13E+09	mm^4
Deflection coefficient:	K =	0.080	
Area of tension steel	A_s =	1611	mm^2
Area of compression steel	A'_s =	402	mm^2
Tension steel over support	As =	1787	mm^2

Material Properties

Concrete strength (28 days)	f_{cu} =	30	MPa	
Concrete elastic modulus at loading	E_c =	21.4	GPa	
Reinforcement elastic modulus	E_s =	200	GPa	
Beam restraint characteristics		Unrestrained		
Cracking moment of element	M_{cr} =	22.9	kNm	
Modulus of rupture (restrained)	f_r =	1.12	MPa	$f_r = 0{,}30\sqrt{f_{cu}}$
Modulus of rupture (unrestrained)	f_r =	2.43	MPa	$f_r = 0{,}65\sqrt{f_{cu}}$

Loads and moments

Permament load on beam (UDL)	G_k =	30	kN/m
Imposed load on beam (UDL)	Q_k =	25	kN/m
Percentage of permanent imposed load	% =	80	
Max moment at midspan	M =	108.428	kNm
Max hogging moment over support	M =	150.67	kNm

Moment of inertia of cracked transformed section: $I_{cr} = \frac{bx^3}{3} + \alpha_e A_s (d-x)^2$ where $x = \alpha_e \rho \left[\sqrt{1 + \frac{2}{\alpha_e \rho}} - 1 \right] d$ = 161 mm

Moment of inertia of cracked section at midspan: I_{cr} = 1.25E+09 mm^4

Moment of inertia of cracked section over supports I_{cr} = 1.34E+09 mm^4

$\alpha_e = \frac{E_s}{E_c}$ and $\rho = \frac{A_s}{bd}$

f_{cu} at age t $f_{cu,t}$ = 13.9 MPa (see explanation below)

Cracking moment: $M_{cr} = \frac{f_r I_g}{y_t}$ = 22.9 kNm

Continuous beam: x at suppor 167 mm

Effective second moment of area: $I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$ = 1.30E+09 mm^4

Concrete Elastic Modulus at loading: $E_{c,28} = K_0 + \alpha f_{cu,28}$ where $f_{cu} \geq 20MPa$

K_0 = 24

α = 0.25

$E_{c,28}$ = 31.5 GPa

$E_{c,t}$ = 21.4 GPa $E_{c,t} = E_{c,28}(0.4 + 0.6 f_{cu,t}/f_{cu,28})$

$\Delta_i = K M_a \frac{l^2}{E_c I_e}$ = 7.8 mm

From SANS and Fultons

Aggregate type	Range of design values			
	3 - 28 days		6 months or older	
	K_0	α	K_0	α
Granite	21	0.25	34	0.1
Greywacke	24	0.25	31	0.2
TM quartzite	23	0.25	34	0.15

Since SANS does not provide an expression to calculate the concrete strength at age t, the Eurocode expression is used:

$\beta_{cc(t)} = \exp\left\{s\left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\}$ = 0.60 where s = 0.25 Mean cylinder strength: f_{cm} = 32 Mpa

$f_{cm(t)} = \beta_{cc(t)} f_{cm}$

$f_{cm(t)}$ = 19.1 MPa (Cylinder strength)

$f_{ck}(t) = f_{cm}(t) - 8$ = 11.1 MPa $(3 < t < 28\ days)$

$f_{cu,t}$ = 13.9 MPa (Cube strength)

Immediate Deflection at ages 3 days - 14 days and 28 days													
	3	4	5	6	7	8	9	10	11	12	13	14	28
$f_{cu,t}$ =	13.9	16.5	18.4	19.9	21.2	22.2	23.0	23.8	24.5	25.1	25.6	26.1	30.0
f_r =	2.43	2.64	2.79	2.90	2.99	3.06	3.12	3.17	3.22	3.25	3.29	3.32	3.56
M_{cr} =	22.9	25.0	26.4	27.4	28.3	28.9	29.5	30.0	30.4	30.7	31.1	31.4	33.6
$E_{c,t}$ =	21.4	23.0	24.2	25.2	25.9	26.6	27.1	27.6	28.0	28.4	28.7	29.0	31.5
I_e =	1.31E+09	1.25E+09	1.21E+09	1.18E+09	1.15E+09	1.14E+09	1.12E+09	1.11E+09	1.10E+09	1.09E+09	1.08E+09	1.08E+09	1.02E+09
Δ =	7.8	7.6	7.4	7.3	7.2	7.2	7.1	7.1	7.0	7.0	7.0	6.9	6.7

Long term creep deflection $\Delta_\infty = \lambda\Delta_i = (1 + x_i\Phi)\Delta_i$

x_i = 0.401 ($x_i = \frac{x}{d}$)

Effective thickness = $\frac{2A}{Exposed\ perimeter}$ = 214 mm

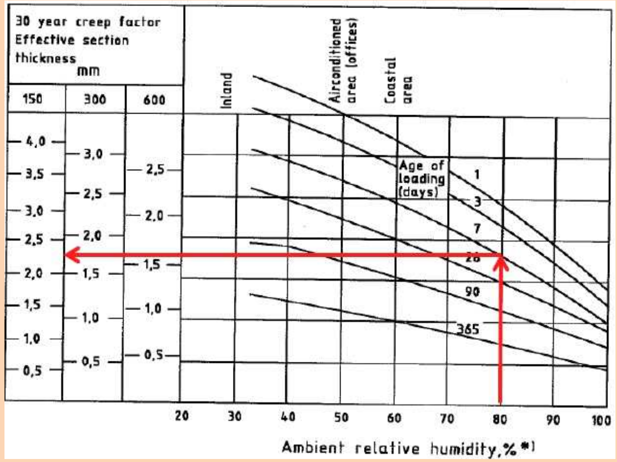
	Min	Max	
Ambient relative humidity =	30	90	%
Age of loading =	3	14	days

LT Creep Deflection

Creep coefficient: Φ = See creep coefficient sheet

Long term creep deflection: Δ_∞ = 18.7 (Also see table below)

Creep coefficients according to SANS 10100-1



RH (%)	Long term creep deflection (mm) for age of loading range 1-14 days													
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
30	40.0	37.3	35.1	33.3	31.7	31.0	30.4	29.8	29.3	28.8	28.4	28.0	27.7	27.3
40	38.9	36.1	34.0	32.2	30.6	29.9	29.2	28.7	28.2	27.8	27.4	27.0	26.6	26.3
50	37.4	34.7	32.6	30.8	29.2	28.5	27.9	27.4	26.9	26.5	26.1	25.8	25.5	25.1
60	35.7	33.0	30.9	29.1	27.5	26.9	26.3	25.9	25.4	25.1	24.7	24.4	24.1	23.8
70	33.6	31.0	29.0	27.2	25.6	25.0	24.5	24.1	23.8	23.4	23.1	22.8	22.6	22.3
80	31.3	28.8	26.7	25.0	23.4	22.9	22.5	22.2	21.9	21.6	21.3	21.1	20.9	20.7
90	28.6	26.2	24.2	22.5	21.0	20.6	20.3	20.0	19.8	19.5	19.3	19.2	19.0	18.9
100	25.7	23.4	21.5	19.8	18.3	18.0	17.8	17.6	17.4	17.3	17.2	17.1	17.0	16.9
110	22.4	20.3	18.5	16.8	15.4	15.2	15.1	15.0	14.9	14.9	14.8	14.8	14.8	14.7

Total Long Term deflection:

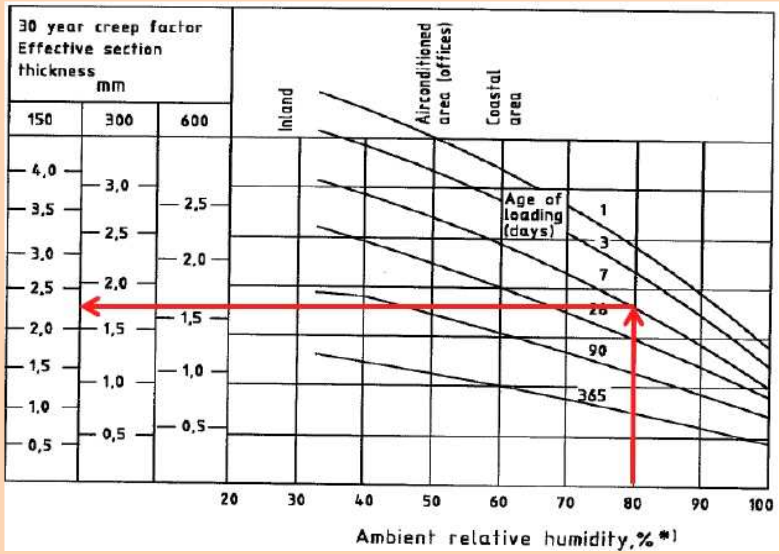
Total Long Term deflection:	Δ_{∞}	=	19.5	mm

RH (%)	Total long term deflection (mm) due to creep and shrinakge, for age of loading range 1-14 days													
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
30	41.1	38.3	36.2	34.4	32.8	32.0	31.4	30.8	30.3	29.9	29.4	29.1	28.7	28.4
40	39.9	37.2	35.0	33.2	31.6	30.9	30.3	29.7	29.2	28.8	28.4	28.0	27.7	27.3
50	38.4	35.7	33.5	31.7	30.1	29.5	28.9	28.4	27.9	27.5	27.1	26.7	26.4	26.1
60	36.6	33.9	31.8	30.0	28.4	27.8	27.2	26.8	26.3	26.0	25.6	25.3	25.0	24.7
70	34.5	31.9	29.8	28.0	26.4	25.9	25.4	25.0	24.6	24.3	23.9	23.7	23.4	23.2
80	32.0	29.5	27.5	25.7	24.1	23.7	23.3	22.9	22.6	22.3	22.0	21.8	21.6	21.4
90	29.1	26.7	24.7	23.0	21.5	21.1	20.8	20.5	20.3	20.0	19.8	19.7	19.5	19.3
100	25.7	23.5	21.5	19.9	18.4	18.1	17.9	17.7	17.5	17.4	17.2	17.1	17.0	16.9
110	21.9	19.8	17.9	16.3	14.8	14.7	14.6	14.5	14.4	14.4	14.3	14.3	14.2	14.2

Age	150	300	600
1	4.2027	3.0288	2.8018
3	3.7875	3.0692	2.527
7	2.9644	2.4223	1.9743
28	2.566	1.9898	1.6987
90	1.9651	1.5551	1.3321

x1	3	3	3
y1	3.7875	3.0692	2.527
x2	7	7	7
y2	2.9644	2.4223	1.9743
y_cr_age	3.7875	3.0692	2.527

x1	150
y1	3.7875
Creep Coeffic.	3.483



Shrinkage deflection according to SANS

Shrinkage deflection: $\Delta_{cs} = K_{sh} k_{cs} \frac{\varepsilon_{sh} L^2}{h}$

Uncracked members: $k_{cs} = 0.7 \sqrt{\rho\left(1 - \frac{\rho'}{\rho}\right)}$ limited to $0.0 \leq k_{cs} \geq 1.0$

Fully cracked members: $k_{cs} = 1 - \frac{\rho'}{\rho}\left[1 - 0.11(3 - \rho)^2\right]$ limited to $0.3 \leq k_{cs} \leq 1.0$

with $\rho = \frac{100 A_s}{bd} \leq 3.0$ and $\rho' = \frac{100 A_s'}{bd}$ limited to $\rho'/\rho \leq 1.0$

Free shrinkage strain = ε_{cs} (Refer to Figure C2)

Effective thickness: 227 mm

$\rho =$ 0.536
$\rho' =$ 0.135

$k_{cs} =$ 0.443

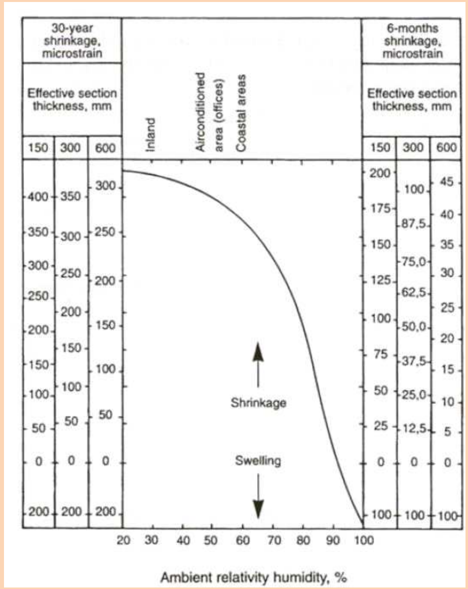
Beam Type = Simply supported

K_{sh} = 0.125

Beam Type	K_{sh}
Cantilever	0.5
Simply supported	0.125
One end continuous	0.086
Both ends continuo	0.063

Shrinkage Microstrain from SANS Figure C2						
	6 month shrinkage			30 year shrinkage		
RH	**150**	**300**	**600**	**150**	**300**	**600**
20%	200	107	48	460	400	325
30%	200	106	47	450	390	315
40%	190	103	45	425	375	305
50%	180	97	43	400	350	290
60%	165	89	39	370	325	270
70%	145	78	33	325	280	230
80%	100	53	23	220	190	160
90%	10	5	2.5	25	20	15
100%	-110	-110	-110	-220	-220	-220

hrinkage strain for sample sectic		
	Duration	
RH	**6 month**	**30 year**
20%	152	429
30%	152	419
40%	145	399
50%	137	374
60%	126	347
70%	111	302
80%	76	205
90%	7	22
100%	-110	-220



Shrinkage deflection		
	6 month	30 year
RH	Δ_{cs} (mm)	Δ_{cs}(mm)
20	0.4	1.0
30	0.4	1.0
40	0.3	0.9
50	0.3	0.9
60	0.3	0.8
70	0.3	0.7
80	0.2	0.5
90	0.0	0.1
100	-0.3	-0.5

60	0.3	0.8

EUROCODE: DEFLECTION CALCULATION OF UNIFORMLY LOADED, SIMPLY SUPPORTED RECTANGULAR BEAM

Beam Geometry and Section Properties

Effective span of member:	$L =$	5000	mm
Section width:	$b =$	280	mm
Overall section depth:	$h =$	450	mm
Concrete cover depth	$c =$	30	mm
Shear reinforcement diamete	$Ø_{Asv} =$	10	mm
Tension rebar diameter	$Ø_{As} =$	20	mm
Compression rebar diameter	$Ø_{Asc} =$	12	mm
Depth to tension rebar	$d =$	400	mm
Depth to compression rebar	$d_2 =$	46	mm
Moment of inertia of section	$I_g =$	2.13E+09	mm^4
Deflection coefficient	$K =$	0.080	
Area of tension steel	$A_s =$	1611	mm²
Area of compression steel	$A_{s2} =$	402	mm²

Applied moments

Max moment (SLS)	$M_{QP} =$	108.4	kNm

Material Properties

Concrete strength (cube strength)	$f_{cu} =$	30	MPa	
Reinforcement elastic modulus	$E_s =$	200	GPa	
Characteristic concrete cylinder strength	$f_{ck} =$	24	MPa	
Concrete mean compressive strength	$f_{cm} =$	32	MPa	$f_{cm} = f_{ck} + 8$
Concrete mean tensile strength	$f_{ctm} =$	3.0	MPa	
Concrete 28 day secant modulus	$E_{cm} =$	31.2	GPa	$E_{cm} = 22[(f_{ck}+8)/10]^{0.3}$
Concrete 28 day tangent modulus	$E_{c28} =$	32.7	GPa	
Long term elastic modulus	$E_{ff} =$	7.2	GPa	$E_{c,eff} = \frac{E_{c28}}{1+\varphi(\infty, t_0)}$
Time of loading (t_0 or t)	$t_0 =$	3	days	
Relative humidity	$RH =$	60	%	
Cement type used in concrete	CEM42.5N			
Creep coefficient	$\varphi(\infty, t_0) =$	3.54		
Effective modulus ratio	$\alpha_e =$	7.5		
Secant modulus for ages other than 28 days	$E_{cm(t)} =$	26.7	GPa	$E_{cm(t)} = \left(f_{cm(t)}/f_{cm}\right)^{0.3} E_{cm}$
Age of concrete adjustment coefficient	$\beta_{cc(t)} =$	0.598		$\beta_{cc(t)} = exp\left\{s\left[1-\left(\frac{28}{t}\right)^{0.5}\right]\right\}$
Coefficient depending on cement type	$s =$	0.25		
Mean concrete compressive stength at age t(	$f_{cm(t)} =$	19.1	MPa	$f_{cm(t)} = \beta_{cc(t)} f_{cm}$
Concrete tensile strength at time t (days)	$f_{ctm(t)} =$	1.8	MPa	$f_{ctm(t)} = \left(\beta_{cc(t)}\right)^{\alpha} f_{ctm}$

Long term deflection $\delta_{QP} = KL^2 \frac{1}{r_{t,QP}}$

Uncracked section properties:

Depth to neutral axis: $x_u = \frac{\frac{bh^2}{2} + (\alpha_e - 1)(A_s d + A_{s2} d_2)}{bh + (\alpha_e - 1)(A_s + A_{s2})}$ = 234.8 mm

Second moment of area $I_u = \frac{bh^3}{12} + bh\left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1)[A_s(d - x_u)^2 + A_{s2}(x_u - d_2)^2]$ = 2.52E+09 mm^4

Cracked section properties:

Depth to neutral axis $x_c = \left\{\left[\left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right)^2 + 2b\left(A_s d\alpha_e + A_{s2}d_2(\alpha_e - 1)\right)\right]^{0.5} - \left(A_s\alpha_e + A_{s2}(\alpha_e - 1)\right)\right\}/b$ = 142.7 mm

Second moment of area: $I_c = \frac{bx_c^3}{3} + \alpha_e A_s(d - x_c)^2 + (\alpha_e - 1)A_{s2}(x_c - d_2)^2$ = 1.09E+09 mm^4

Cracking moment $M_{cr} = \frac{0.9 f_{ctm} I_u}{h - x_u}$ = 19.0 kNm

$I_c = \frac{bx_c^3}{3} + \alpha_e A_s(d - x_c)^2 + (\alpha_e - 1)A_{s2}(d_2 - x_c)^2$

Degree of cracking $\zeta = 1 - 0.5\left(M_{cr}/M_{QP}\right)^2$ = 0.985

(if $M_{cr} > M_{QP}$ then $\zeta = 0$)

Degree of cracking (short term $\zeta = 1 - \left(M_{cr}/M_{QP}\right)^2$ = 0.969

Curvature due to flexure and shrinkage strain:

Flexural curvature: $\frac{1}{r_n} = \zeta\frac{M_{QP}}{E_{eff} I_c} + (1 - \zeta)\frac{M_{QP}}{E_{eff} I_u}$ = 3.64E-06

Flexural curvature (incl. creep $\frac{1}{r_n} =$ .36E-05

Curvature due to shrinkage strain $\frac{1}{r_{cs}} = \zeta \varepsilon_{cs} \alpha_e \frac{S_c}{I_c} + (1-\zeta)\varepsilon_{cs}\alpha_e \frac{S_u}{I_u}$ = 5.13E-09

where $S_u = A_s(d - x_u) - A_{s2}(x_u - d_2)$ = 190264.6 mm³

$S_c = A_s(d - x_c) - A_{s2}(x_c - d_2)$ = 375695.5 mm³

ε_{cs} = 4.19E-04

Total curvature: $\frac{1}{r_{t,QP}} = \frac{1}{r_n} + \frac{1}{r_{cs}}$ = 1.36E-05 mm^{-1}

Immediate deflection: Δ= 7.3 mm

Long term deflection δ_{QP} 27.2 mm

Immediate Deflection at ages 3 days - 14 days, 21 & 28 days														
	3	4	5	6	7	8	9	10	11	12	13	14	21	28
$\beta_{cc(t)}$ =	0.598	0.663	0.711	0.748	0.779	0.804	0.826	0.845	0.862	0.876	0.890	0.902	0.962	1.000
$f_{cm(t)}$ =	19.1	21.2	22.7	23.9	24.9	25.7	26.4	27.0	27.6	28.0	28.5	28.9	30.8	32.0
$E_{cm(t)}$ =	26.7	27.6	28.1	28.6	28.9	29.2	29.5	29.7	29.8	30.0	30.1	30.2	30.8	31.2
α_e =	7.5	7.3	7.1	7.0	6.9	6.8	6.8	6.7	6.7	6.7	6.6	6.6	6.5	6.4
x_u =	234.79	234.48	234.27	234.12	234.00	233.91	233.83	233.77	233.71	233.67	233.62	233.59	233.41	233.30
I_u =	2.52E+09	2.50E+09	2.49E+09	2.49E+09	2.48E+09	2.48E+09	2.48E+09	2.47E+09	2.47E+09	2.47E+09	2.47E+09	2.47E+09	2.46E+09	2.45E+09
x_c =	142.67	141.10	140.03	139.24	138.63	138.14	137.73	137.39	137.09	136.84	136.61	136.41	135.43	134.85
I_c =	1.09E+09	1.07E+09	1.05E+09	1.04E+09	1.03E+09	1.02E+09	1.02E+09	1.01E+09	1.01E+09	1.00E+09	9.99E+08	9.96E+08	9.81E+08	9.72E+08
$f_{ctm(t)}$ =	1.8	2.0	2.1	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.7	2.7	2.9	3.0
M_{cr} =	19.0	20.9	22.4	23.5	24.4	25.1	25.8	26.3	26.8	27.2	27.6	28.0	29.7	30.8
ζ =	0.969	0.963	0.957	0.953	0.950	0.946	0.944	0.941	0.939	0.937	0.935	0.934	0.925	0.919
Δ =	7.3	7.2	7.1	7.1	7.1	7.0	7.0	7.0	7.0	6.9	6.9	6.9	6.8	6.8

Long term deflection														
	Age of loading (days)													
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E_{ff} (20%)	5.5	5.7	5.9	6.1	6.3	6.4	6.5	6.6	6.7	6.8	6.9	6.9	7.0	7.1
E_{ff} (30%)	5.8	6.1	6.3	6.5	6.6	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.4	7.5
E_{ff} (40%)	6.2	6.5	6.7	6.9	7.1	7.2	7.3	7.5	7.6	7.7	7.7	7.8	7.9	8.0
E_{ff} (50%)	6.7	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.1	8.2	8.3	8.4	8.4	8.5
E_{ff} (60%)	7.2	7.5	7.8	8.0	8.1	8.3	8.4	8.6	8.7	8.8	8.9	9.0	9.1	9.1
E_{ff} (70%)	7.8	8.2	8.4	8.6	8.8	9.0	9.1	9.2	9.4	9.5	9.6	9.7	9.8	9.9
E_{ff} (80%)	8.6	8.9	9.2	9.4	9.6	9.8	9.9	10.1	10.2	10.3	10.4	10.5	10.6	10.7
E_{ff} (90%)	9.6	10.0	10.2	10.5	10.7	10.9	11.0	11.2	11.3	11.4	11.5	11.7	11.7	11.8
E_{ff} (100%)	10.7	11.1	11.4	11.6	11.8	12.0	12.2	12.3	12.5	12.6	12.7	12.8	12.9	13.0
α_e	7.5	7.3	7.1	7.0	6.9	6.8	6.8	6.7	6.7	6.7	6.6	6.6	6.5	6.4
x_u	234.8	234.5	234.3	234.1	234.0	233.9	233.8	233.8	233.7	233.7	233.6	233.6	233.4	233.3
I_u	2.52E+09	2.50E+09	2.49E+09	2.49E+09	2.48E+09	2.48E+09	2.48E+09	2.47E+09	2.47E+09	2.47E+09	2.47E+09	2.47E+09	2.46E+09	2.45E+09
x_k	142.7	141.1	140.0	139.2	138.6	138.1	137.7	137.4	137.1	136.8	136.6	136.4	135.4	134.9
I_k	1.09E+09	1.07E+09	1.05E+09	1.04E+09	1.03E+09	1.02E+09	1.02E+09	1.01E+09	1.01E+09	1.00E+09	9.99E+08	9.96E+08	9.81E+08	9.72E+08
M_{cr}	19.0	20.9	22.4	23.5	24.4	25.1	25.8	26.3	26.8	27.2	27.6	28.0	29.7	30.8
ζ	0.985	0.981	0.979	0.977	0.975	0.973	0.972	0.971	0.969	0.968	0.968	0.967	0.962	0.960
$1/r_n$ 20%	1.79E-05	1.75E-05	1.71E-05	1.68E-05	1.66E-05	1.63E-05	1.61E-05	1.60E-05	1.58E-05	1.56E-05	1.55E-05	1.54E-05	1.52E-05	1.50E-05
$1/r_n$ 30%	1.68E-05	1.64E-05	1.61E-05	1.58E-05	1.56E-05	1.54E-05	1.52E-05	1.50E-05	1.49E-05	1.47E-05	1.46E-05	1.45E-05	1.43E-05	1.42E-05
$1/r_n$ 40%	1.57E-05	1.54E-05	1.51E-05	1.49E-05	1.47E-05	1.45E-05	1.43E-05	1.41E-05	1.40E-05	1.39E-05	1.37E-05	1.36E-05	1.35E-05	1.33E-05
$1/r_n$ 50%	1.47E-05	1.44E-05	1.41E-05	1.39E-05	1.37E-05	1.35E-05	1.34E-05	1.32E-05	1.31E-05	1.30E-05	1.29E-05	1.28E-05	1.26E-05	1.25E-05
$1/r_n$ 60%	1.36E-05	1.33E-05	1.31E-05	1.29E-05	1.27E-05	1.26E-05	1.24E-05	1.23E-05	1.22E-05	1.21E-05	1.20E-05	1.19E-05	1.18E-05	1.16E-05
$1/r_n$ 70%	1.26E-05	1.23E-05	1.21E-05	1.19E-05	1.18E-05	1.16E-05	1.15E-05	1.14E-05	1.13E-05	1.12E-05	1.11E-05	1.10E-05	1.09E-05	1.08E-05
$1/r_n$ 80%	1.15E-05	1.13E-05	1.11E-05	1.09E-05	1.08E-05	1.07E-05	1.06E-05	1.05E-05	1.04E-05	1.03E-05	1.02E-05	1.01E-05	1.00E-05	9.92E-06
$1/r_n$ 90%	1.03E-05	1.01E-05	9.94E-06	9.82E-06	9.70E-06	9.60E-06	9.51E-06	9.43E-06	9.35E-06	9.29E-06	9.22E-06	9.16E-06	9.06E-06	8.97E-06
$1/r_n$ 100%	9.22E-06	9.08E-06	8.97E-06	8.86E-06	8.77E-06	8.68E-06	8.61E-06	8.54E-06	8.48E-06	8.42E-06	8.37E-06	8.31E-06	8.23E-06	8.15E-06

S_u	190265	190890	191308	191612	191845	192032	192186	192315	192426	192522	192606	192681	193042	193254
S_c	375695	378867	381021	382606	383834	384822	385638	386328	386921	387438	387893	388298	390264	391431
$1/r_{cr}$ 20%	4.86E-09	5.23E-09	5.52E-09	5.75E-09	5.96E-09	6.13E-09	6.28E-09	6.42E-09	6.55E-09	6.66E-09	6.77E-09	6.87E-09	6.94E-09	7.01E-09
$1/r_{cr}$ 30%	5.08E-09	5.46E-09	5.76E-09	6.00E-09	6.21E-09	6.39E-09	6.55E-09	6.69E-09	6.82E-09	6.94E-09	7.05E-09	7.15E-09	7.22E-09	7.29E-09
$1/r_{cr}$ 40%	5.23E-09	5.62E-09	5.92E-09	6.17E-09	6.38E-09	6.56E-09	6.72E-09	6.87E-09	7.00E-09	7.12E-09	7.23E-09	7.33E-09	7.41E-09	7.48E-09
$1/r_{cr}$ 50%	5.27E-09	5.66E-09	5.96E-09	6.20E-09	6.41E-09	6.59E-09	6.75E-09	6.90E-09	7.03E-09	7.15E-09	7.26E-09	7.36E-09	7.43E-09	7.50E-09
$1/r_{cr}$ 60%	5.13E-09	5.51E-09	5.80E-09	6.03E-09	6.23E-09	6.41E-09	6.56E-09	6.70E-09	6.82E-09	6.93E-09	7.04E-09	7.13E-09	7.20E-09	7.27E-09
$1/r_{cr}$ 70%	4.74E-09	5.08E-09	5.35E-09	5.56E-09	5.74E-09	5.90E-09	6.04E-09	6.16E-09	6.27E-09	6.37E-09	6.47E-09	6.55E-09	6.61E-09	6.67E-09
$1/r_{cr}$ 80%	3.99E-09	4.27E-09	4.49E-09	4.66E-09	4.81E-09	4.94E-09	5.05E-09	5.15E-09	5.25E-09	5.33E-09	5.40E-09	5.47E-09	5.52E-09	5.57E-09
$1/r_{cr}$ 90%	2.73E-09	2.92E-09	3.06E-09	3.18E-09	3.27E-09	3.36E-09	3.43E-09	3.50E-09	3.56E-09	3.61E-09	3.66E-09	3.71E-09	3.74E-09	3.77E-09
$1/r_{cr}$ 100%	6.34E-10	6.76E-10	7.08E-10	7.34E-10	7.56E-10	7.75E-10	7.92E-10	8.06E-10	8.19E-10	8.31E-10	8.42E-10	8.53E-10	8.59E-10	8.66E-10
$1/r_{t,QP}$ 20%	1.79E-05	1.75E-05	1.71E-05	1.68E-05	1.66E-05	1.63E-05	1.61E-05	1.60E-05	1.58E-05	1.56E-05	1.55E-05	1.54E-05	1.52E-05	1.50E-05
$1/r_{t,QP}$ 30%	1.68E-05	1.64E-05	1.61E-05	1.59E-05	1.56E-05	1.54E-05	1.52E-05	1.51E-05	1.49E-05	1.48E-05	1.46E-05	1.45E-05	1.43E-05	1.42E-05
$1/r_{t,QP}$ 40%	1.58E-05	1.54E-05	1.51E-05	1.49E-05	1.47E-05	1.45E-05	1.43E-05	1.41E-05	1.40E-05	1.39E-05	1.37E-05	1.36E-05	1.35E-05	1.33E-05
$1/r_{t,QP}$ 50%	1.47E-05	1.44E-05	1.41E-05	1.39E-05	1.37E-05	1.35E-05	1.34E-05	1.32E-05	1.31E-05	1.30E-05	1.29E-05	1.28E-05	1.26E-05	1.25E-05
$1/r_{t,QP}$ 60%	1.36E-05	1.34E-05	1.31E-05	1.29E-05	1.27E-05	1.26E-05	1.24E-05	1.23E-05	1.22E-05	1.21E-05	1.20E-05	1.19E-05	1.18E-05	1.16E-05
$1/r_{t,QP}$ 70%	1.26E-05	1.23E-05	1.21E-05	1.19E-05	1.18E-05	1.16E-05	1.15E-05	1.14E-05	1.13E-05	1.12E-05	1.11E-05	1.10E-05	1.09E-05	1.08E-05
$1/r_{t,QP}$ 80%	1.15E-05	1.13E-05	1.11E-05	1.09E-05	1.08E-05	1.07E-05	1.06E-05	1.05E-05	1.04E-05	1.03E-05	1.02E-05	1.01E-05	1.00E-05	9.93E-06
$1/r_{t,QP}$ 90%	1.03E-05	1.01E-05	9.95E-06	9.82E-06	9.71E-06	9.61E-06	9.52E-06	9.43E-06	9.36E-06	9.29E-06	9.22E-06	9.16E-06	9.06E-06	8.98E-06
$1/r_{t,QP}$ 100%	9.22E-06	9.08E-06	8.97E-06	8.86E-06	8.77E-06	8.68E-06	8.61E-06	8.54E-06	8.48E-06	8.42E-06	8.37E-06	8.32E-06	8.23E-06	8.15E-06
δ_{QP} 20%	35.7	34.9	34.2	33.7	33.1	32.7	32.3	31.9	31.6	31.3	31.0	30.7	30.3	30.0
δ_{QP} 30%	33.6	32.9	32.2	31.7	31.2	30.8	30.4	30.1	29.8	29.5	29.2	29.0	28.6	28.3
δ_{QP} 40%	31.5	30.8	30.2	29.7	29.3	28.9	28.6	28.3	28.0	27.7	27.5	27.3	26.9	26.6
δ_{QP} 50%	29.4	28.8	28.2	27.8	27.4	27.0	26.7	26.4	26.2	25.9	25.7	25.5	25.2	24.9
δ_{QP} 60%	27.2	26.7	26.2	25.8	25.5	25.2	24.9	24.6	24.4	24.2	24.0	23.8	23.5	23.3
δ_{QP} 70%	25.1	24.6	24.2	23.9	23.5	23.3	23.0	22.8	22.6	22.4	22.2	22.1	21.8	21.6
δ_{QP} 80%	22.9	22.5	22.2	21.8	21.6	21.3	21.1	20.9	20.7	20.6	20.4	20.3	20.0	19.8
δ_{QP} 90%	20.5	20.2	19.9	19.6	19.4	19.2	19.0	18.9	18.7	18.6	18.4	18.3	18.1	17.9
δ_{QP} 100%	18.4	18.2	17.9	17.7	17.5	17.4	17.2	17.1	16.9	16.8	16.7	16.6	16.4	16.3

Creep coefficient according to Eurocode Annex B

$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0)$

where $\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$ and $\beta_c(t,t_0) = \left[\frac{(t-t_0)}{(\beta_H + t - t_0)}\right]^{0.3}$

Relative humidty effects: $\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}}$ for $f_{cm} \leq 35MPa$

f_{cm} = 32 Mpa
RH = 60 %

$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \cdot \alpha_1\right]\alpha_2$ for $f_{cm} \geq 35$

Cement type	α
CEM42.5N	0

Concrete strength effects $\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$ = 2.97

Concrete age of loading effects: $\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})}$ = 0.725

Effects of cement type $t_0 = t_{0,T} \cdot \left(\frac{9}{2 + t_{0,T}^{1.2}} + 1\right)^{\alpha} \geq 0.5$ = 3

Notional member size: $h_0 = \frac{2A_c}{u}$ 213.6 mm

Coefficients for concrete strength: $\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7}$ = 1.065 $\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2}$ = 1.018 $\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5}$ = 1.046

Relative humidity coefficient: $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500$ for $f_{cm} \leq 35$

Relative humidity coefficient $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3$ for $f_{cm} \geq 35$

	Change in Relative Humidity (RH) in %								
	20	30	40	50	60	70	80	90	100
β_H $f_{cm} \leq 35$	570.34	570.34	570.34	570.37	571.21	584.23	723.97	1500.00	1500.00
β_H $f_{cm} \geq 35$	581.80	581.80	581.80	581.83	582.66	595.68	735.43	1568.74	1568.74

Temperature adjusted age of concrete: $t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i$ = 3.4 days

Temperature during period: $T(\Delta t_i)$ = 23 °C

Number of days where temperature prevails (Δt_i) 3 days

Age of concrete considered: 10950 days

φ_{RH} = 1.67

φ_0 = 3.59

β_H = 571.205

$\beta_c(t, t_0)$ = 0.985

$\varphi(t, t_0)$ = 3.54

RH effects	Change in Relative Humidity (RH) in %									
	20	30	40	50	60	70	80	90	100	1500.00
φ_{RH}	2.34	2.17	2.00	1.84	1.67	1.50	1.33	1.17	1.00	
$\beta(f_{cm})$	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	
β_H	570.34	570.34	570.34	570.37	571.21	584.23	723.97	1500.00	1500.00	

	Age of loading (days)													
	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$t_T = t_{0,T}$	3.4	4.6	5.7	6.9	8.0	9.2	10.3	11.5	12.6	13.8	14.9	16.0	17.2	18.3
t_0	3	5	6	7	8	9	10	11	13	14	15	16	17	18
$\beta(t_0)$	0.725	0.687	0.659	0.637	0.619	0.603	0.590	0.578	0.568	0.559	0.551	0.543	0.536	0.529

	RH	Change in creep coefficient at age of loading (days) for given effective thickness at different RH values													
		3	4	5	6	7	8	9	10	11	12	13	14	15	16
φ_0	20%	5.032	4.770	4.575	4.422	4.296	4.189	4.098	4.017	3.946	3.881	3.823	3.770	3.721	3.676
φ_0	30%	4.672	4.428	4.248	4.105	3.988	3.890	3.804	3.730	3.663	3.604	3.550	3.500	3.455	3.413
φ_0	40%	4.312	4.087	3.920	3.789	3.681	3.590	3.511	3.442	3.381	3.326	3.276	3.230	3.189	3.150
φ_0	50%	3.952	3.746	3.593	3.473	3.374	3.290	3.218	3.155	3.099	3.048	3.003	2.961	2.922	2.887
φ_0	60%	3.592	3.405	3.266	3.156	3.066	2.991	2.925	2.868	2.816	2.771	2.729	2.691	2.656	2.624
φ_0	70%	3.232	3.063	2.939	2.840	2.759	2.691	2.632	2.580	2.534	2.493	2.455	2.421	2.390	2.361
φ_0	80%	2.872	2.722	2.611	2.524	2.452	2.391	2.339	2.293	2.252	2.215	2.182	2.152	2.124	2.098
φ_0	90%	2.512	2.381	2.284	2.207	2.144	2.091	2.046	2.005	1.970	1.938	1.908	1.882	1.857	1.835
φ_0	100%	2.152	2.040	1.957	1.891	1.837	1.792	1.752	1.718	1.687	1.660	1.635	1.612	1.591	1.572

$\beta_c(t,t_0)$	20%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	30%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	40%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	50%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	60%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	70%	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985
$\beta_c(t,t_0)$	80%	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981
$\beta_c(t,t_0)$	90%	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962
$\beta_c(t,t_0)$	100%	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962

$\varphi(t, t_0)$	20%	4.96	4.70	4.51	4.35	4.23	4.13	4.04	3.96	3.89	3.82	3.77	3.71	3.66	3.62
$\varphi(t, t_0)$	30%	4.60	4.36	4.18	4.04	3.93	3.83	3.75	3.67	3.61	3.55	3.50	3.45	3.40	3.36
$\varphi(t, t_0)$	40%	4.25	4.03	3.86	3.73	3.63	3.54	3.46	3.39	3.33	3.28	3.23	3.18	3.14	3.10
$\varphi(t, t_0)$	50%	3.89	3.69	3.54	3.42	3.32	3.24	3.17	3.11	3.05	3.00	2.96	2.92	2.88	2.84
$\varphi(t, t_0)$	60%	3.54	3.35	3.22	3.11	3.02	2.95	2.88	2.82	2.77	2.73	2.69	2.65	2.62	2.58
$\varphi(t, t_0)$	70%	3.18	3.02	2.89	2.80	2.72	2.65	2.59	2.54	2.49	2.45	2.42	2.38	2.35	2.32
$\varphi(t, t_0)$	80%	2.82	2.67	2.56	2.48	2.41	2.35	2.29	2.25	2.21	2.17	2.14	2.11	2.08	2.06
$\varphi(t, t_0)$	90%	2.42	2.29	2.20	2.12	2.06	2.01	1.97	1.93	1.90	1.86	1.84	1.81	1.79	1.77
$\varphi(t, t_0)$	100%	2.07	1.96	1.88	1.82	1.77	1.72	1.69	1.65	1.62	1.60	1.57	1.55	1.53	1.51

Shrinkage strain according to Eurocode

f_{ck} = 24 MPa

h = 450 mm

b = 280 mm

$h_0 = \frac{2A_c}{u}$ = 214 mm

k_h = 0.836

RH = 60 %

t = 10950 days (Age of concrete at age considered)

t_s = 3 days (Age of concrete at the beginning of drying shrinkage)

f_{cm} = 32 MPa

f_{cmo} = 10 MPa

h_0	k_h
100	1.00
200	0.85
300	0.75
500	0.70

RH_0 = 100 %

Drying shrinkage strain:

$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}$ = 3.84E-04

$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}}$ = 0.989

$\varepsilon_{cd,0} = 0.85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH}$ = 4.64E-04

$\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right]$ = 1.22

Cement	α_{ds1}	α_{ds2}
CEM42.5N	4	0.12

	Basic drying shrinkage strain ($\varepsilon_{cd,0}$) for different RH values								
RH (%)	20	30	40	50	60	70	80	90	100
RH_0 (%)	100	100	100	100	100	100	100	100	100
β_{RH}	1.54	1.51	1.45	1.36	1.22	1.02	0.76	0.42	0.00
f_{cm}	32	32	32	32	32	32	32	32	32
f_{cmo}	10	10	10	10	10	10	10	10	10
α_{ds1}	4	4	4	4	4	4	4	4	4
α_{ds2}	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
$\varepsilon_{cd,0}$ =	5.88E-04	5.76E-04	5.54E-04	5.18E-04	4.64E-04	3.89E-04	2.89E-04	1.61E-04	0.00E+00

Autogenous shrinkage strain:

$\beta_{as}(t) = 1 - exp(-0.2t^{0.5})$ = 1.000

$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10)10^{-6}$ = 3.5E-05

$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty)$ = 3.50E-05

Total shrinkage strain

$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$ = 4.19E-04

		5 Year shrinkage strain at different ages of loading and relative humidity values													
t		10950	10950	10950	10950	10950	10950	10950	10950	10950	10950	10950	10950	10950	10950
t_s		1	2	3	4	5	6	7	7	7	7	7	7	7	7
$\beta_{ds}(t,t_s)$		0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989
$\varepsilon_{cd}(t)$	20%	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04	4.86E-04
$\varepsilon_{cd}(t)$	30%	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04	4.77E-04
$\varepsilon_{cd}(t)$	40%	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04	4.58E-04
$\varepsilon_{cd}(t)$	50%	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04	4.29E-04
$\varepsilon_{cd}(t)$	60%	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04	3.84E-04
$\varepsilon_{cd}(t)$	70%	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04	3.22E-04
$\varepsilon_{cd}(t)$	80%	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04	2.39E-04
$\varepsilon_{cd}(t)$	90%	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04	1.33E-04
$\varepsilon_{cd}(t)$	100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\beta_{as}(t)$		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\varepsilon_{ca}(t)$		3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05
ε_{cs}	20%	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04	5.21E-04
ε_{cs}	30%	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04	5.12E-04
ε_{cs}	40%	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04	4.93E-04
ε_{cs}	50%	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04	4.64E-04
ε_{cs}	60%	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04	4.19E-04
ε_{cs}	70%	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04	3.57E-04
ε_{cs}	80%	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04	2.74E-04
ε_{cs}	90%	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04	1.68E-04
ε_{cs}	100%	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05	3.50E-05

ACI: DEFLECTION CALCULATION OF UNIFORMLY LOADED, SIMPLY SUPPORTED RECTANGULAR BEAM

Beam Geometry and Section Properties

Effective span of member:	L =	5000	mm
Section width:	b =	280	mm
Overall section depth:	h =	450	mm
Concrete cover depth	c =	30	mm
Shear reinforcement diamete	$Ø_{Asv}$ =	10	mm
Tension rebar diameter	$Ø_{As}$ =	20	mm
Compression rebar diameter	$Ø_{Asc}$ =	12	mm
Depth to tension rebar	d =	400	mm
Depth to compression rebar	d' =	46	mm
Moment of inertia of section	I_g =	2.13E+09	mm^4
Deflection coefficient	K =	0.080	
Area of tension steel	A_s =	1611	mm^2
Area of compression steel	A'_s =	402	mm^2
Tension steel over support	As =	1787	mm^2

Material Properties

Concrete strength (cylinder strength)	f'_c =	24	MPa
Reinforcement elastic modulus	E_s =	200	GPa
Reinforcement yield strength	f_y =	450	MPa
Unit weight of concrete	w_c =	2325	kg/m^3
Concrete modulus of rupture	f_r =	3.1	MPa
Concrete elastic modulus at loading	E_c =	24	GPa
Long term elastic modulus	E_{ff} =	9.1	GPa
Time of loading (t_0 or t)	t_0 =	3	days
Relative humidity	RH =	60	%
Cement type used in concrete (S, N or R	CEM42.5N		
Creep coefficient	$\phi(t, t_0)$	1.61	
Effective modulus ratio (28 day strength)	α_e	8.5	

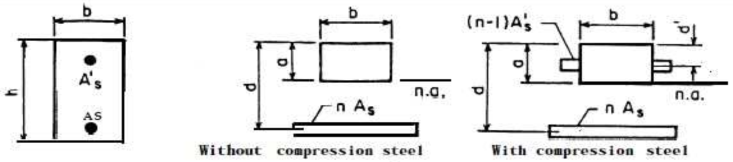
$f_r = 0.623\sqrt{f_c}$

Loads and moments

Permament load on beam (UDL)	G_k =	30	kN/m
Imposed load on beam (UDL)	Q_k =	25	kN/m
Percentage of permanent imposed load	% =	80	
Max moment at midspan	M_a =	108.4	kNm
Max hogging moment over support	M =	150.7	kNm

Initial deflection = $\Delta = KL^2 \frac{M}{E_c I_e}$ (Note: Continuous beams! Calculate average effective moment of inertia from max. positive and negative moment sections

Moment of inertia of cracked transformed section:



$B = b/(nA_s)$ $r = (n-1)A_s'/(nA_s)$ $I_g = bh^3/12$

Without compression steel
$a = (\sqrt{2dB+1} - 1)/B$
$I_{cr} = ba^3/3 + nA_s(d-a)^2$

With compression steel
$a = [\sqrt{2dB(1+rd'/d) + (1+r)^2} - (1+r)]/B$
$I_{cr} = ba^3/3 + nA_s(d-a)^2 + (n-1)A_s'(a-d')^2$

(a) Rectangular Sections

$$I_{cr} = \frac{bx^3}{3} + \alpha_e A_s(d-x)^2 + (\alpha_e - 1)A'_s(x-d')^2$$

$x = [\{2dB(1+rd'/d) + (1+r)^2\}^{0.5} - (1+r)]/B$ = 170.23 mm (positive)
176.93 mm (negative)

$B = b/(\alpha_e A_s)$ = 0.014 (Positive) 0.013 (Negative)

$r = (\alpha_e - 1)A'_s/\alpha_e A_s$ = 0.230 (Positive) 0.207 (Negative)

$\therefore I_{cr} =$ 1.60E+09 mm^4 (Positive) 1.71E+09 (Negative)

Cracking moment: $M_{cr} = \frac{f_r I_g}{y_t}$ = 19.5 kNm

$f_r = 0.623\sqrt{f_c'}$ = 2.07 MPa

$f_{cmt} = \left[\frac{t}{a+bt}\right] f_{cm28}$ = 11.0 Mpa

Type of cement	Moist-cured	
	a	b
Type I	4.00	0.85

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$ = 1.65E+09 mm^4 (Note: Average effective moment of inertia)

Table A.4—Values of the constant *a* and *b* for use in Eq. (A-17), ACI 209R-92 model

Type of cement	Moist-cured concrete		Steam-cured concrete	
	a	b	a	b
I	4.0	0.85	1.0	0.95
III	2.3	0.92	0.70	0.98

$E_c = w_c^{1.5} \cdot 0.043\sqrt{f_c'}$ = 16.0 GPa

Initial deflection: Δ = 8.2 mm

Total long term deflection due to creep and shrinkag = Δ_l 18.6 mm

Simplified long term deflection factor $\lambda_\Delta = \frac{\xi}{1+50\rho'}$

$\lambda_\Delta =$ 1.70

Duration	ξ
> 5 years	2.0
12 months	1.4
6 months	1.2
3 months	1.0

Simplified long term deflection: $\Delta_l = \lambda_\Delta \times \Delta_i$

Long-term Deflection	Duration			
	3 months	6 months	12 months	> 5 years
λ_Δ	0.85	1.02	1.19	1.70
Δ_l (mm)	6.9	8.3	9.7	13.9

The following table shows the change in immediate deflection at different ages of loading according to the calculation method as shown above.

Immediate Deflection at ages 3 days - 14 days, 21 & 28 days														
Age (days)	3	4	5	6	7	8	9	10	11	12	13	14	21	28
f_{cmt} =	11.0	13.0	14.5	15.8	16.9	17.8	18.5	19.2	19.8	20.3	20.7	21.1	23.1	24.2
E_c =	16.0	17.4	18.4	19.2	19.8	20.3	20.8	21.1	21.4	21.7	21.9	22.2	23.2	23.7
α_e =	12.5	11.5	10.9	10.4	10.1	9.8	9.6	9.5	9.3	9.2	9.1	9.0	8.6	8.4
f_r =	2.07	2.24	2.38	2.48	2.56	2.63	2.68	2.73	2.77	2.81	2.84	2.86	2.99	3.06
M_{cr} =	19.5	21.2	22.5	23.4	24.2	24.8	25.4	25.8	26.2	26.5	26.8	27.1	28.3	28.9
I_e =	1.65E+09	1.56E+09	1.50E+09	1.46E+09	1.42E+09	1.40E+09	1.38E+09	1.36E+09	1.35E+09	1.33E+09	1.32E+09	1.31E+09	1.28E+09	1.25E+09
$\Delta =$	8.2	8.0	7.9	7.8	7.7	7.6	7.6	7.5	7.5	7.5	7.5	7.4	7.3	7.3

Long term deflection (>5 years) at age of loading 3 days - 14 days, 21 days & 28 days using simplified method														
λ_Δ	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
Δ_l (mm)	13.9	13.6	13.3	13.2	13.0	12.9	12.9	12.8	12.7	12.7	12.7	12.6	12.5	12.4

Total long term deflection (More detailed approach as specified in ACI)													
Deflection Type	Relative Humidity	Age of loading											
		3	4	5	6	7	8	9	10	11	12	13	14
Initial Δ		8.2	8.0	7.9	7.8	7.7	7.6	7.6	7.5	7.5	7.5	7.5	7.4
Creep Deflection	20%	11.9	11.9	11.9	11.7	11.5	11.4	11.2	11.1	11.0	10.9	10.4	10.1
	30%	11.2	11.2	11.2	11.0	10.8	10.7	10.6	10.5	10.4	10.3	9.8	9.5
	40%	10.5	10.5	10.5	10.3	10.2	10.0	9.9	9.8	9.7	9.6	9.2	8.9
	50%	9.8	9.8	9.8	9.6	9.5	9.4	9.3	9.2	9.1	9.0	8.6	8.3
	60%	9.1	9.1	9.1	8.9	8.8	8.7	8.6	8.5	8.4	8.4	8.0	7.7
	70%	8.4	8.4	8.4	8.2	8.1	8.0	7.9	7.9	7.8	7.7	7.3	7.1
	80%	7.7	7.7	7.7	7.5	7.4	7.4	7.3	7.2	7.1	7.1	6.7	6.5
	90%	7.0	7.0	7.0	6.9	6.8	6.7	6.6	6.5	6.5	6.4	6.1	5.9
	100%	6.3	6.3	6.3	6.2	6.1	6.0	5.9	5.9	5.8	5.8	5.5	5.3
Shrinkage Deflection	20%	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
	30%	1.4	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2
	40%	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1
	50%	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	60%	1.0	1.0	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
	70%	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
	80%	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
	90%	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	100%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total Deflection	20%	21.6	21.4	21.2	20.9	20.6	20.4	20.2	20.0	19.9	19.8	19.2	18.9
	30%	20.8	20.6	20.4	20.0	19.8	19.6	19.4	19.3	19.1	19.0	18.5	18.2
	40%	20.0	19.7	19.6	19.2	19.0	18.8	18.6	18.5	18.4	18.3	17.8	17.4
	50%	19.1	18.9	18.8	18.4	18.2	18.0	17.9	17.7	17.6	17.5	17.0	16.7
	60%	18.3	18.1	17.9	17.6	17.4	17.2	17.1	16.9	16.8	16.7	16.3	16.0
	70%	17.5	17.3	17.1	16.8	16.6	16.4	16.3	16.2	16.1	16.0	15.6	15.3
	80%	16.7	16.4	16.3	16.0	15.8	15.7	15.5	15.4	15.3	15.2	14.9	14.6
	90%	15.6	15.4	15.2	15.0	14.8	14.7	14.5	14.4	14.3	14.2	13.9	13.7
	100%	14.5	14.3	14.2	13.9	13.8	13.6	13.5	13.4	13.3	13.3	13.0	12.8

Long term creep deflection according to ACI318

Long term deflection: $\Delta_l = k_r \cdot \phi \cdot \Delta_i$

Redcution factor: $k_r = \dfrac{0.85}{(1 + 50 * \rho')}$ = 0.72

Creep coefficient: $\phi(t, t_0) = \dfrac{(t - t_0)^{\Psi}}{d + (t - t_0)^{\Psi}} \phi_u$ where $\Psi = 1.0$ and $d = f = 26.0e^{[1.42\times10^{-2}(V/S)]}$ = 89 days

Ultimate creep coefficient: $\phi_u = 2.35\gamma_c$

Correction factor: $\gamma_c = \gamma_{c,t0}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\Psi}\gamma_{sh,\alpha}$

Moist curing factor: $\gamma_{c,t0} = 1.25{t_0}^{-0.118}$ = 1.00 (Curing duration is limited to a minimum of 7 days)

Relative humidity factor: $\gamma_{c,RH} = 1.27 - 0.67h$ = 0.87

Volume : Surface Area ratio $\gamma_{c,vs} = \frac{2}{3}\left(1 + 1.13e^{\{-0.0213(V/S)\}}\right)$ = 0.79

Slump factor: $\gamma_{c,s} = 0.82 + 0.00264s$ = 1.018

Fine aggregate : total aggregate factor $\gamma_{c,\Psi} = 0.88 + 0.0024\Psi$ = 0.992

Air content factor: $\gamma_{c,\alpha} = 0.46 + 0.09\alpha \geq 1$ $\gamma_{c,\alpha}$ = 1

γ_c = 0.69

ϕ_u = 1.62

$\phi(t, t_0)$ = 1.607

Δ_l = **9.5** mm

Correction factor		Moist curing duration (days)												
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**21**
Curing	$\gamma_{c,t0}$	1.00	1.00	1.00	1.00	1.00	0.98	0.96	0.95	0.94	0.93	0.92	0.92	0.87
Humidity	$\gamma_{c,RH}$ 20%	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
	$\gamma_{c,RH}$ 30%	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
	$\gamma_{c,RH}$ 40%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$\gamma_{c,RH}$ 50%	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
	$\gamma_{c,RH}$ 60%	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
	$\gamma_{c,RH}$ 70%	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80
	$\gamma_{c,RH}$ 80%	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73
	$\gamma_{c,RH}$ 90%	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	$\gamma_{c,RH}$ 100%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Member size	$\gamma_{c,vs}$	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787	0.787
Slump	$\gamma_{c,s}$	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018	1.018
Fine aggregate	$\gamma_{c,\Psi}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Air content	$\gamma_{c,\alpha}$	1	1	1	1	1	1	1	1	1	1	1	1	1
Cumulative	γ_c 20%	0.90	0.90	0.90	0.90	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.79
	γ_c 30%	0.85	0.85	0.85	0.85	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.74
	γ_c 40%	0.80	0.80	0.80	0.80	0.80	0.78	0.77	0.76	0.75	0.74	0.73	0.73	0.69
	γ_c 50%	0.74	0.74	0.74	0.74	0.74	0.73	0.72	0.71	0.70	0.69	0.69	0.68	0.65
	γ_c 60%	0.69	0.69	0.69	0.69	0.69	0.67	0.66	0.66	0.65	0.64	0.64	0.63	0.60
	γ_c 70%	0.64	0.64	0.64	0.64	0.64	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.56
	γ_c 80%	0.58	0.58	0.58	0.58	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.53	0.51
	γ_c 90%	0.53	0.53	0.53	0.53	0.53	0.52	0.51	0.50	0.50	0.49	0.49	0.48	0.46
	γ_c 100%	0.48	0.48	0.48	0.48	0.48	0.47	0.46	0.45	0.45	0.44	0.44	0.44	0.42
Ultimate creep	ϕ_u 20%	2.12	2.12	2.12	2.12	2.12	2.07	2.04	2.02	2.00	1.98	1.96	1.94	1.85
	ϕ_u 30%	2.00	2.00	2.00	2.00	2.00	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.74
	ϕ_u 40%	1.87	1.87	1.87	1.87	1.87	1.83	1.80	1.78	1.76	1.74	1.73	1.71	1.63
	ϕ_u 50%	1.74	1.74	1.74	1.74	1.74	1.71	1.68	1.66	1.64	1.63	1.61	1.60	1.52
	ϕ_u 60%	1.62	1.62	1.62	1.62	1.62	1.58	1.56	1.54	1.53	1.51	1.50	1.48	1.41
	ϕ_u 70%	1.49	1.49	1.49	1.49	1.49	1.46	1.44	1.42	1.41	1.39	1.38	1.37	1.30
	ϕ_u 80%	1.37	1.37	1.37	1.37	1.37	1.34	1.32	1.30	1.29	1.28	1.27	1.25	1.20
	ϕ_u 90%	1.24	1.24	1.24	1.24	1.24	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.09
	ϕ_u 100%	1.12	1.12	1.12	1.12	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.03	0.98

Long term creep deflection for different RH values and curing durations														
Creep coefficie	Days	RH	Curing Duration											
			3	4	5	6	7	8	9	10	11	12	13	14	21
$\phi(t,t_0)$ (3 Month)	90	20%	1.051	1.051	1.051	1.051	1.051	1.028	1.013	1.001	0.990	0.980	0.970	0.962	0.917
		30%	0.989	0.989	0.989	0.989	0.989	0.967	0.954	0.942	0.931	0.922	0.913	0.905	0.863
		40%	0.927	0.927	0.927	0.927	0.927	0.906	0.894	0.883	0.873	0.864	0.856	0.848	0.809
		50%	0.865	0.865	0.865	0.865	0.865	0.846	0.834	0.824	0.815	0.806	0.799	0.792	0.755
		60%	0.803	0.803	0.803	0.803	0.803	0.785	0.774	0.765	0.756	0.748	0.741	0.735	0.701
		70%	0.741	0.741	0.741	0.741	0.741	0.725	0.715	0.706	0.698	0.691	0.684	0.678	0.647
		80%	0.679	0.679	0.679	0.679	0.679	0.664	0.655	0.647	0.639	0.633	0.627	0.622	0.592
		90%	0.617	0.617	0.617	0.617	0.617	0.603	0.595	0.588	0.581	0.575	0.570	0.565	0.538
		100%	0.555	0.555	0.555	0.555	0.555	0.543	0.535	0.529	0.523	0.517	0.513	0.508	0.484
$\phi(t,t_0)$ (6 Month)	180	20%	1.413	1.413	1.413	1.413	1.413	1.382	1.363	1.346	1.331	1.317	1.305	1.294	1.233
		30%	1.330	1.330	1.330	1.330	1.330	1.301	1.283	1.267	1.253	1.240	1.228	1.217	1.161
		40%	1.246	1.246	1.246	1.246	1.246	1.219	1.202	1.187	1.174	1.162	1.151	1.141	1.088
		50%	1.163	1.163	1.163	1.163	1.163	1.137	1.122	1.108	1.096	1.084	1.074	1.065	1.015
		60%	1.080	1.080	1.080	1.080	1.080	1.056	1.041	1.029	1.017	1.007	0.997	0.989	0.942
		70%	0.996	0.996	0.996	0.996	0.996	0.974	0.961	0.949	0.939	0.929	0.920	0.912	0.870
		80%	0.913	0.913	0.913	0.913	0.913	0.893	0.881	0.870	0.860	0.851	0.843	0.836	0.797
		90%	0.830	0.830	0.830	0.830	0.830	0.811	0.800	0.790	0.782	0.774	0.766	0.760	0.724
		100%	0.746	0.746	0.746	0.746	0.746	0.730	0.720	0.711	0.703	0.696	0.689	0.683	0.651
$\phi(t,t_0)$ (12 Month)	365	20%	1.703	1.703	1.703	1.703	1.703	1.666	1.643	1.623	1.604	1.588	1.573	1.559	1.487
		30%	1.603	1.603	1.603	1.603	1.603	1.568	1.546	1.527	1.510	1.494	1.480	1.467	1.399
		40%	1.502	1.502	1.502	1.502	1.502	1.469	1.449	1.431	1.415	1.401	1.388	1.376	1.311
		50%	1.402	1.402	1.402	1.402	1.402	1.371	1.352	1.336	1.321	1.307	1.295	1.284	1.224
		60%	1.302	1.302	1.302	1.302	1.302	1.273	1.255	1.240	1.226	1.213	1.202	1.192	1.136
		70%	1.201	1.201	1.201	1.201	1.201	1.175	1.158	1.144	1.131	1.120	1.109	1.100	1.048
		80%	1.101	1.101	1.101	1.101	1.101	1.076	1.062	1.048	1.037	1.026	1.016	1.008	0.961
		90%	1.000	1.000	1.000	1.000	1.000	0.978	0.965	0.953	0.942	0.932	0.924	0.916	0.873
		100%	0.900	0.900	0.900	0.900	0.900	0.880	0.868	0.857	0.847	0.839	0.831	0.824	0.785

		20%	2.022	2.022	2.022	2.022	2.022	1.977	1.950	1.926	1.904	1.885	1.867	1.851	1.765	1.706
		30%	1.903	1.903	1.903	1.903	1.903	1.861	1.835	1.812	1.792	1.774	1.757	1.742	1.660	1.605
		40%	1.783	1.783	1.783	1.783	1.783	1.744	1.720	1.699	1.680	1.663	1.647	1.633	1.556	1.504
$\phi(t,t_0)$		50%	1.664	1.664	1.664	1.664	1.664	1.627	1.605	1.585	1.567	1.551	1.537	1.523	1.452	1.404
(5 Years)	1825	60%	1.545	1.545	1.545	1.545	1.545	1.511	1.490	1.472	1.455	1.440	1.427	1.414	1.348	1.303
		70%	1.426	1.426	1.426	1.426	1.426	1.394	1.375	1.358	1.343	1.329	1.317	1.305	1.244	1.203
		80%	1.306	1.306	1.306	1.306	1.306	1.278	1.260	1.244	1.230	1.218	1.206	1.196	1.140	1.102
		90%	1.187	1.187	1.187	1.187	1.187	1.161	1.145	1.131	1.118	1.107	1.096	1.087	1.036	1.001
		100%	1.068	1.068	1.068	1.068	1.068	1.044	1.030	1.017	1.006	0.996	0.986	0.978	0.932	0.901
		20%	6.2	6.2	6.2	6.2	6.2	6.1	6.0	5.9	5.8	5.8	5.7	5.7	5.4	5.2
		30%	5.8	5.8	5.8	5.8	5.8	5.7	5.6	5.6	5.5	5.4	5.4	5.3	5.1	4.9
		40%	5.5	5.5	5.5	5.5	5.5	5.4	5.3	5.2	5.2	5.1	5.1	5.0	4.8	4.6
Δ_l		50%	5.1	5.1	5.1	5.1	5.1	5.0	4.9	4.9	4.8	4.8	4.7	4.7	4.5	4.3
(3 Month)	90	60%	4.7	4.7	4.7	4.7	4.7	4.6	4.6	4.5	4.5	4.4	4.4	4.3	4.1	4.0
		70%	4.4	4.4	4.4	4.4	4.4	4.3	4.2	4.2	4.1	4.1	4.0	4.0	3.8	3.7
		80%	4.0	4.0	4.0	4.0	4.0	3.9	3.9	3.8	3.8	3.7	3.7	3.7	3.5	3.4
		90%	3.6	3.6	3.6	3.6	3.6	3.6	3.5	3.5	3.4	3.4	3.4	3.3	3.2	3.1
		100%	3.3	3.3	3.3	3.3	3.3	3.2	3.2	3.1	3.1	3.1	3.0	3.0	2.9	2.8
		20%	8.3	8.3	8.3	8.3	8.3	8.2	8.1	8.0	7.9	7.8	7.7	7.6	7.3	7.0
		30%	7.9	7.9	7.9	7.9	7.9	7.7	7.6	7.5	7.4	7.3	7.3	7.2	6.9	6.6
		40%	7.4	7.4	7.4	7.4	7.4	7.2	7.1	7.0	6.9	6.9	6.8	6.7	6.4	6.2
Δ_l		50%	6.9	6.9	6.9	6.9	6.9	6.7	6.6	6.5	6.5	6.4	6.3	6.3	6.0	5.8
(6 Month)	180	60%	6.4	6.4	6.4	6.4	6.4	6.2	6.2	6.1	6.0	5.9	5.9	5.8	5.6	5.4
		70%	5.9	5.9	5.9	5.9	5.9	5.8	5.7	5.6	5.5	5.5	5.4	5.4	5.1	5.0
		80%	5.4	5.4	5.4	5.4	5.4	5.3	5.2	5.1	5.1	5.0	5.0	4.9	4.7	4.5
		90%	4.9	4.9	4.9	4.9	4.9	4.8	4.7	4.7	4.6	4.6	4.5	4.5	4.3	4.1
		100%	4.4	4.4	4.4	4.4	4.4	4.3	4.3	4.2	4.2	4.1	4.1	4.0	3.8	3.7
		20%	10.1	10.1	10.1	10.1	10.1	9.8	9.7	9.6	9.5	9.4	9.3	9.2	8.8	8.5
		30%	9.5	9.5	9.5	9.5	9.5	9.3	9.1	9.0	8.9	8.8	8.7	8.7	8.3	8.0
		40%	8.9	8.9	8.9	8.9	8.9	8.7	8.6	8.5	8.4	8.3	8.2	8.1	7.7	7.5
Δ_l		50%	8.3	8.3	8.3	8.3	8.3	8.1	8.0	7.9	7.8	7.7	7.6	7.6	7.2	7.0
(12 Month)	365	60%	7.7	7.7	7.7	7.7	7.7	7.5	7.4	7.3	7.2	7.2	7.1	7.0	6.7	6.5
		70%	7.1	7.1	7.1	7.1	7.1	6.9	6.8	6.8	6.7	6.6	6.6	6.5	6.2	6.0
		80%	6.5	6.5	6.5	6.5	6.5	6.4	6.3	6.2	6.1	6.1	6.0	6.0	5.7	5.5
		90%	5.9	5.9	5.9	5.9	5.9	5.8	5.7	5.6	5.6	5.5	5.5	5.4	5.2	5.0
		100%	5.3	5.3	5.3	5.3	5.3	5.2	5.1	5.1	5.0	5.0	4.9	4.9	4.6	4.5

Δ_l (5 Years)	1825	20%	11.9	11.9	11.9	11.9	11.9	11.7	11.5	11.4	11.2	11.1	11.0	10.9	10.4	10.1
		30%	11.2	11.2	11.2	11.2	11.2	11.0	10.8	10.7	10.6	10.5	10.4	10.3	9.8	9.5
		40%	10.5	10.5	10.5	10.5	10.5	10.3	10.2	10.0	9.9	9.8	9.7	9.6	9.2	8.9
		50%	9.8	9.8	9.8	9.8	9.8	9.6	9.5	9.4	9.3	9.2	9.1	9.0	8.6	8.3
		60%	9.1	9.1	9.1	9.1	9.1	8.9	8.8	8.7	8.6	8.5	8.4	8.4	8.0	7.7
		70%	8.4	8.4	8.4	8.4	8.4	8.2	8.1	8.0	7.9	7.9	7.8	7.7	7.3	7.1
		80%	7.7	7.7	7.7	7.7	7.7	7.5	7.4	7.4	7.3	7.2	7.1	7.1	6.7	6.5
		90%	7.0	7.0	7.0	7.0	7.0	6.9	6.8	6.7	6.6	6.5	6.5	6.4	6.1	5.9
		100%	6.3	6.3	6.3	6.3	6.3	6.2	6.1	6.0	5.9	5.9	5.8	5.8	5.5	5.3

Shrinkage defelction according to ACI 318

Shrinkage deflection: $\Delta_{cs} = K_{sh} L^2 \frac{1}{r_{cs}}$ where shrinkage deflection coefficient $K_{sh} =$ 0.09 (for simple beam) No. of spans: ' 4

Shrinkage curvature: $\frac{1}{r_{cs}} = 0.7 \frac{\varepsilon_{cs}}{h} (\rho - \rho')^{1/3} \left[\frac{\rho - \rho'}{\rho}\right]^2$

Support condition	K_{sh}	
Cantilever	0.500	
Simply supported	0.125	
Both ends continuous	0.09	*
* Two-span continuous beam	0.084	
Continuous Beam (3 or more spans)	0.090	End Span
	0.065	Interior Span

Tension reinforcement ratio: $\rho = 100A_s/bd$ = 1.44

Compression reinforcement ratio: $\rho' = 100A_s'/bd$ = 0.36

Shrinkage strain:

$\varepsilon_{cs}(t,t_c) = \frac{(t-t_c)^\alpha}{f + (t-t_c)^\alpha} \times \varepsilon_{shu}$ where $\alpha = 1$ and $f = 26.0e^{\{1.42\times10^{-2}(V/S)\}}$ = 89 days

$\varepsilon_{shu} = 780 \times 10^{-6} \gamma_{sh}$ and $\gamma_{sh} = \gamma_{sh,tc}\gamma_{sh,RH}\gamma_{sh,vs}\gamma_{sh,s}\gamma_{sh,\Psi}\gamma_{sh,c}\gamma_{sh,\alpha}$

Initial moist curing coefficient: $\gamma_{sh,tc} = 1.202 - 0.2337\log(t_c)$

$\gamma_{sh,tc}$ = 1.09

Ambient relative humidity coefficient: $\gamma_{sh,RH} = \begin{cases} 1.40 - 1.02h \text{ for } 0.40 \le h \le 0.80 \\ 3.00 - 3.0h \text{ for } 0.80 \le h \le 1 \end{cases}$

$\gamma_{sh,RH}$ = 0.8

Moist curing duration (days)	$\gamma_{sh,tc}$
1	1.20
3	1.09
7	1.00
14	0.93
28	0.86
90	0.75

RH	$\gamma_{sh,RH}$
20%	1.2
30%	1.1
40%	1.0
50%	0.9
60%	0.8
70%	0.7
80%	0.6
90%	0.3
100%	0.0

Member size factor:

$\gamma_{sh,vs} = 1.2e^{\{-0.00472(V/S)\}}$

$\gamma_{sh,vs} =$ 0.798

Cement content factor:

$\gamma_{sh,c} = 0.75 + 0.00061c$

$\gamma_{sh,c} =$ 0.8476

Air content factor:

$\gamma_{sh,\alpha} = 0.95 + 0.008\alpha \geq 1$

$\gamma_{sh,\alpha} =$ 1

Slump factor:

$\gamma_{sh,s} = 0.89 + 0.00161s$

$\gamma_{sh,s} =$ 1.01075

Fine aggregate content coefficient: $\gamma_{sh,\Psi} = \begin{cases} 0.30 + 0.014\Psi \; for \; \Psi \leq 50\% \\ 0.90 + 0.002\Psi \; for \; \Psi > 50\% \end{cases}$

$\gamma_{sh,\Psi}$ = 0.99

Ψ	Ratio, fine aggregate to total
30%	0.72
30%	0.72
40%	0.86
50%	1.00
60%	1.02
70%	1.04
80%	1.06
90%	1.08
100%	1.10

γ_{sh} = 0.58

ε_{shu} = 0.0004554

$\varepsilon_{cs}(t, t_c)$ = 0.0004517

$\frac{1}{r_{cs}}$ = 4.06E-07

Δ_{cs} = 0.9 mm

Correction factor			Moist curing duration (days)										
			3	4	5	6	7	8	9	10	11	12	13
Curing	$\gamma_{sh,tc}$		1.09	1.06	1.04	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.94
Humidity	$\gamma_{sh,RH}$	20%	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20
	$\gamma_{sh,RH}$	30%	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
	$\gamma_{sh,RH}$	40%	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	$\gamma_{sh,RH}$	50%	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
	$\gamma_{sh,RH}$	60%	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
	$\gamma_{sh,RH}$	70%	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
	$\gamma_{sh,RH}$	80%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
	$\gamma_{sh,RH}$	90%	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	$\gamma_{sh,RH}$	00%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Member size	$\gamma_{sh,vs}$		0.798	0.798	0.798	0.798	0.798	0.798	0.798	0.798	0.798	0.798	0.798
Slump	$\gamma_{sh,s}$		1.01075	1.01075	1.01075	1.01075	1.01075	1.01075	1.01075	1.01075	1.01075	1.01075	1.01075
Fine aggregate	$\gamma_{sh,\Psi}$		0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524
Air content	$\gamma_{sh,\alpha}$		1	1	1	1	1	1	1	1	1	1	1
Cumulative	γ_{sh}	20%	1.00	0.98	0.95	0.94	0.92	0.91	0.90	0.89	0.88	0.87	0.87
	γ_{sh}	30%	0.92	0.89	0.87	0.86	0.84	0.83	0.82	0.81	0.81	0.80	0.79
	γ_{sh}	40%	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.72
	γ_{sh}	50%	0.75	0.73	0.71	0.70	0.69	0.68	0.67	0.66	0.66	0.65	0.64
	γ_{sh}	60%	0.66	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.57
	γ_{sh}	70%	0.58	0.56	0.55	0.54	0.53	0.52	0.52	0.51	0.51	0.50	0.50
	γ_{sh}	80%	0.50	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.43
	γ_{sh}	90%	0.25	0.24	0.24	0.24	0.23	0.23	0.23	0.22	0.22	0.22	0.22
	γ_{sh}	100%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ultimate shrinkage strain	ε_{shu}	20%	7.82E-04	7.61E-04	7.45E-04	7.32E-04	7.20E-04	7.11E-04	7.02E-04	6.94E-04	6.87E-04	6.81E-04	6.75E-04
	ε_{shu}	30%	7.15E-04	6.96E-04	6.81E-04	6.69E-04	6.59E-04	6.50E-04	6.42E-04	6.35E-04	6.29E-04	6.23E-04	6.18E-04
	ε_{shu}	40%	6.49E-04	6.31E-04	6.18E-04	6.07E-04	5.97E-04	5.89E-04	5.82E-04	5.76E-04	5.70E-04	5.65E-04	5.60E-04
	ε_{shu}	50%	5.82E-04	5.66E-04	5.54E-04	5.44E-04	5.36E-04	5.29E-04	5.22E-04	5.17E-04	5.12E-04	5.07E-04	5.02E-04
	ε_{shu}	60%	5.15E-04	5.01E-04	4.91E-04	4.82E-04	4.75E-04	4.68E-04	4.63E-04	4.57E-04	4.53E-04	4.49E-04	4.45E-04
	ε_{shu}	70%	4.49E-04	4.37E-04	4.27E-04	4.20E-04	4.13E-04	4.08E-04	4.03E-04	3.98E-04	3.94E-04	3.91E-04	3.87E-04
	ε_{shu}	80%	3.92E-04	3.82E-04	3.74E-04	3.67E-04	3.61E-04	3.56E-04	3.52E-04	3.48E-04	3.45E-04	3.42E-04	3.39E-04
	ε_{shu}	90%	1.96E-04	1.91E-04	1.87E-04	1.83E-04	1.81E-04	1.78E-04	1.76E-04	1.74E-04	1.72E-04	1.71E-04	1.69E-04
	ε_{shu}	100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Long term shrinkage deflection for different RH values and curing durations									
Shrinkage strain	Days	RH	Curing Duration						
			3	4	5	6	7	8	9
$\varepsilon_{cs}(t,t_c)$	90	20%	3.88E-04	3.75E-04	3.65E-04	3.56E-04	3.48E-04	3.42E-04	3.35E-04
		30%	3.54E-04	3.43E-04	3.34E-04	3.26E-04	3.19E-04	3.13E-04	3.07E-04
		40%	3.21E-04	3.11E-04	3.03E-04	2.95E-04	2.89E-04	2.83E-04	2.78E-04
		50%	2.88E-04	2.79E-04	2.71E-04	2.65E-04	2.59E-04	2.54E-04	2.50E-04
		60%	2.55E-04	2.47E-04	2.40E-04	2.35E-04	2.30E-04	2.25E-04	2.21E-04
		70%	2.22E-04	2.15E-04	2.09E-04	2.04E-04	2.00E-04	1.96E-04	1.92E-04
		80%	1.94E-04	1.88E-04	1.83E-04	1.79E-04	1.75E-04	1.71E-04	1.68E-04
		90%	9.72E-05	9.41E-05	9.15E-05	8.93E-05	8.74E-05	8.57E-05	8.41E-05
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\varepsilon_{cs}(t,t_c)$	180	20%	5.21E-04	5.06E-04	4.95E-04	4.85E-04	4.76E-04	4.69E-04	4.63E-04
		30%	4.77E-04	4.63E-04	4.52E-04	4.43E-04	4.36E-04	4.29E-04	4.23E-04
		40%	4.32E-04	4.20E-04	4.10E-04	4.02E-04	3.95E-04	3.89E-04	3.84E-04
		50%	3.88E-04	3.77E-04	3.68E-04	3.61E-04	3.55E-04	3.49E-04	3.44E-04
		60%	3.43E-04	3.34E-04	3.26E-04	3.19E-04	3.14E-04	3.09E-04	3.05E-04
		70%	2.99E-04	2.90E-04	2.84E-04	2.78E-04	2.73E-04	2.69E-04	2.65E-04
		80%	2.61E-04	2.54E-04	2.48E-04	2.43E-04	2.39E-04	2.35E-04	2.32E-04
		90%	1.31E-04	1.27E-04	1.24E-04	1.22E-04	1.20E-04	1.18E-04	1.16E-04
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\varepsilon_{cs}(t,t_c)$	365	20%	6.28E-04	6.11E-04	5.98E-04	5.87E-04	5.77E-04	5.69E-04	5.62E-04
		30%	5.75E-04	5.59E-04	5.47E-04	5.37E-04	5.28E-04	5.21E-04	5.14E-04
		40%	5.21E-04	5.07E-04	4.96E-04	4.87E-04	4.79E-04	4.72E-04	4.66E-04
		50%	4.68E-04	4.55E-04	4.45E-04	4.37E-04	4.30E-04	4.24E-04	4.18E-04
		60%	4.14E-04	4.03E-04	3.94E-04	3.87E-04	3.80E-04	3.75E-04	3.70E-04
		70%	3.60E-04	3.51E-04	3.43E-04	3.37E-04	3.31E-04	3.27E-04	3.22E-04
		80%	3.15E-04	3.07E-04	3.00E-04	2.94E-04	2.90E-04	2.86E-04	2.82E-04
		90%	1.58E-04	1.53E-04	1.50E-04	1.47E-04	1.45E-04	1.43E-04	1.41E-04
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

$\varepsilon_{cs}(t,t_c)$	1825	20%	7.46E-04	7.26E-04	7.10E-04	6.98E-04	6.87E-04	6.78E-04	6.69E-04
		30%	6.82E-04	6.64E-04	6.50E-04	6.38E-04	6.28E-04	6.20E-04	6.12E-04
		40%	6.19E-04	6.02E-04	5.89E-04	5.79E-04	5.70E-04	5.62E-04	5.55E-04
		50%	5.55E-04	5.40E-04	5.29E-04	5.19E-04	5.11E-04	5.04E-04	4.98E-04
		60%	4.91E-04	4.78E-04	4.68E-04	4.60E-04	4.53E-04	4.46E-04	4.41E-04
		70%	4.28E-04	4.16E-04	4.07E-04	4.00E-04	3.94E-04	3.89E-04	3.84E-04
		80%	3.74E-04	3.64E-04	3.56E-04	3.50E-04	3.45E-04	3.40E-04	3.36E-04
		90%	1.87E-04	1.82E-04	1.78E-04	1.75E-04	1.72E-04	1.70E-04	1.68E-04
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\frac{1}{r_{cs}}$	90	20%	3.48E-07	3.37E-07	3.28E-07	3.20E-07	3.13E-07	3.07E-07	3.01E-07
		30%	3.19E-07	3.08E-07	3.00E-07	2.93E-07	2.86E-07	2.81E-07	2.76E-07
		40%	2.89E-07	2.79E-07	2.72E-07	2.65E-07	2.60E-07	2.55E-07	2.50E-07
		50%	2.59E-07	2.51E-07	2.44E-07	2.38E-07	2.33E-07	2.28E-07	2.24E-07
		60%	2.29E-07	2.22E-07	2.16E-07	2.11E-07	2.06E-07	2.02E-07	1.99E-07
		70%	2.00E-07	1.93E-07	1.88E-07	1.84E-07	1.80E-07	1.76E-07	1.73E-07
		80%	1.75E-07	1.69E-07	1.64E-07	1.61E-07	1.57E-07	1.54E-07	1.51E-07
		90%	8.74E-08	8.45E-08	8.22E-08	8.03E-08	7.86E-08	7.70E-08	7.56E-08
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\frac{1}{r_{cs}}$	180	20%	4.68E-07	4.55E-07	4.44E-07	4.36E-07	4.28E-07	4.22E-07	4.16E-07
		30%	4.28E-07	4.16E-07	4.07E-07	3.99E-07	3.92E-07	3.86E-07	3.80E-07
		40%	3.89E-07	3.77E-07	3.69E-07	3.61E-07	3.55E-07	3.50E-07	3.45E-07
		50%	3.49E-07	3.39E-07	3.31E-07	3.24E-07	3.19E-07	3.14E-07	3.09E-07
		60%	3.09E-07	3.00E-07	2.93E-07	2.87E-07	2.82E-07	2.78E-07	2.74E-07
		70%	2.69E-07	2.61E-07	2.55E-07	2.50E-07	2.46E-07	2.42E-07	2.38E-07
		80%	2.35E-07	2.28E-07	2.23E-07	2.19E-07	2.15E-07	2.11E-07	2.09E-07
		90%	1.17E-07	1.14E-07	1.11E-07	1.09E-07	1.07E-07	1.06E-07	1.04E-07
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

$\frac{1}{r_{cs}}$	365	20%	5.65E-07	5.49E-07	5.37E-07	5.27E-07	5.19E-07	5.12E-07	5.05E-07
		30%	5.16E-07	5.02E-07	4.91E-07	4.82E-07	4.75E-07	4.68E-07	4.62E-07
		40%	4.68E-07	4.56E-07	4.46E-07	4.37E-07	4.30E-07	4.24E-07	4.19E-07
		50%	4.20E-07	4.09E-07	4.00E-07	3.92E-07	3.86E-07	3.81E-07	3.76E-07
		60%	3.72E-07	3.62E-07	3.54E-07	3.47E-07	3.42E-07	3.37E-07	3.33E-07
		70%	3.24E-07	3.15E-07	3.08E-07	3.02E-07	2.98E-07	2.93E-07	2.90E-07
		80%	2.83E-07	2.76E-07	2.70E-07	2.65E-07	2.60E-07	2.57E-07	2.53E-07
		90%	1.42E-07	1.38E-07	1.35E-07	1.32E-07	1.30E-07	1.28E-07	1.27E-07
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
$\frac{1}{r_{cs}}$	1825	20%	6.70E-07	6.52E-07	6.38E-07	6.27E-07	6.17E-07	6.09E-07	6.02E-07
		30%	6.13E-07	5.97E-07	5.84E-07	5.73E-07	5.65E-07	5.57E-07	5.50E-07
		40%	5.56E-07	5.41E-07	5.29E-07	5.20E-07	5.12E-07	5.05E-07	4.99E-07
		50%	4.99E-07	4.85E-07	4.75E-07	4.67E-07	4.59E-07	4.53E-07	4.48E-07
		60%	4.42E-07	4.30E-07	4.21E-07	4.13E-07	4.07E-07	4.01E-07	3.96E-07
		70%	3.84E-07	3.74E-07	3.66E-07	3.60E-07	3.54E-07	3.49E-07	3.45E-07
		80%	3.36E-07	3.27E-07	3.20E-07	3.14E-07	3.10E-07	3.05E-07	3.02E-07
		90%	1.68E-07	1.64E-07	1.60E-07	1.57E-07	1.55E-07	1.53E-07	1.51E-07
		100%	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Δ_{cs}	90	20%	0.8	0.8	0.7	0.7	0.7	0.7	0.7
		30%	0.7	0.7	0.7	0.7	0.6	0.6	0.6
		40%	0.6	0.6	0.6	0.6	0.6	0.6	0.6
		50%	0.6	0.6	0.5	0.5	0.5	0.5	0.5
		60%	0.5	0.5	0.5	0.5	0.5	0.5	0.4
		70%	0.4	0.4	0.4	0.4	0.4	0.4	0.4
		80%	0.4	0.4	0.4	0.4	0.4	0.3	0.3
		90%	0.2	0.2	0.2	0.2	0.2	0.2	0.2
		100%	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Δ_{cs}	180	20%	1.1	1.0	1.0	1.0	1.0	0.9	0.9
		30%	1.0	0.9	0.9	0.9	0.9	0.9	0.9
		40%	0.9	0.8	0.8	0.8	0.8	0.8	0.8
		50%	0.8	0.8	0.7	0.7	0.7	0.7	0.7
		60%	0.7	0.7	0.7	0.6	0.6	0.6	0.6
		70%	0.6	0.6	0.6	0.6	0.6	0.5	0.5
		80%	0.5	0.5	0.5	0.5	0.5	0.5	0.5
		90%	0.3	0.3	0.3	0.2	0.2	0.2	0.2
		100%	0.0	0.0	0.0	0.0	0.0	0.0	0.0

		20%	1.3	1.2	1.2	1.2	1.2	1.2	1.1
		30%	1.2	1.1	1.1	1.1	1.1	1.1	1.0
		40%	1.1	1.0	1.0	1.0	1.0	1.0	0.9
Δ_{cs}	365	50%	0.9	0.9	0.9	0.9	0.9	0.9	0.8
		60%	0.8	0.8	0.8	0.8	0.8	0.8	0.7
		70%	0.7	0.7	0.7	0.7	0.7	0.7	0.7
		80%	0.6	0.6	0.6	0.6	0.6	0.6	0.6
		90%	0.3	0.3	0.3	0.3	0.3	0.3	0.3
		100%	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Δ_{cs}	1825	20%	1.5	1.5	1.4	1.4	1.4	1.4	1.4
		30%	1.4	1.3	1.3	1.3	1.3	1.3	1.2
		40%	1.3	1.2	1.2	1.2	1.2	1.1	1.1
		50%	1.1	1.1	1.1	1.0	1.0	1.0	1.0
		60%	1.0	1.0	0.9	0.9	0.9	0.9	0.9
		70%	0.9	0.8	0.8	0.8	0.8	0.8	0.8
		80%	0.8	0.7	0.7	0.7	0.7	0.7	0.7
		90%	0.4	0.4	0.4	0.4	0.3	0.3	0.3
		100%	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Appendix E – Screen Shots of Case Studies in Excel Model

Case Study 1: Comparison of deflections at different ages of loading, using calculation procedures as given in design codes

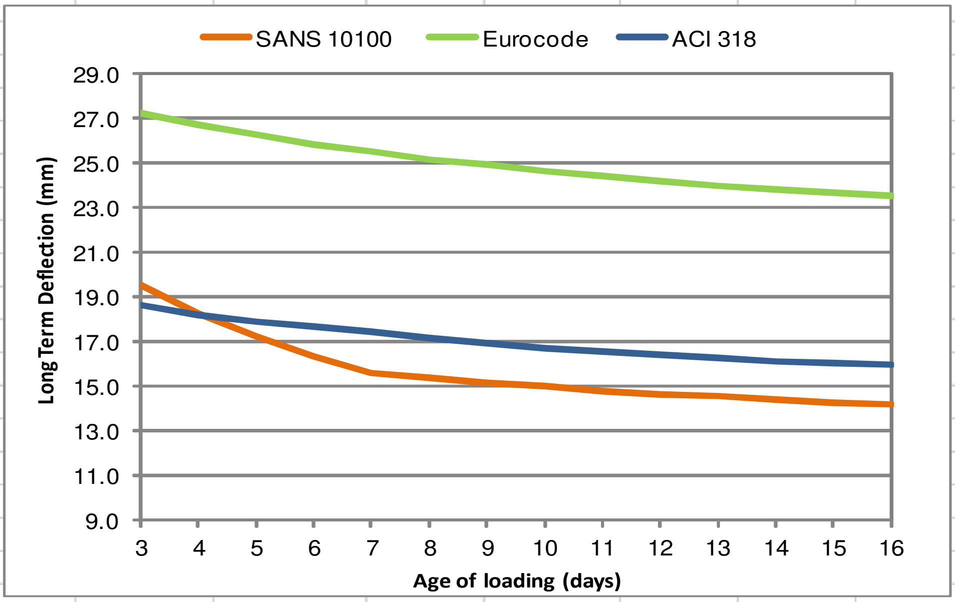
Objective: Comparison of estimated long term deflection values of a continuous RC beam loaded at age 3 - 14 days.

Design input: Identical beam geometry and material properties were used in all calculations. Age of loading is limited to 14 days. Relative humidity of 60% is used in this example.

Max Age (days)	**16**	**Step size (days)**	**1**	**Cement Type**	CEM42.5N
RH (%)	**60**	**Concrete Strength (MPa)**	**30**		

	Long term deflection		
Age of loading (days)	**SANS 10100**	**Eurocode**	**ACI 318**
3	19.5	27.2	18.6
4	18.2	26.7	18.1
5	17.2	26.2	17.8
6	16.3	25.8	17.6
7	15.6	25.5	17.4
8	15.3	25.2	17.1
9	15.1	24.9	16.9
10	14.9	24.6	16.7
11	14.8	24.4	16.5
12	14.6	24.2	16.4
13	14.5	24.0	16.2
14	14.4	23.8	16.1
15	14.3	23.6	16.0
16	14.1	23.5	15.9

Long Term Deflection
Age of loading



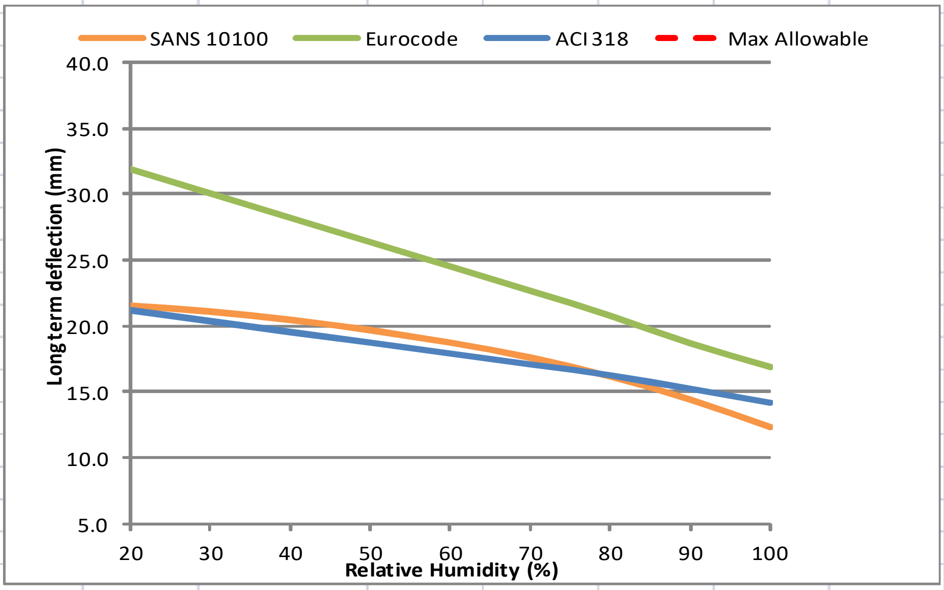
Case Study 2: Comparison of deflections at different relative humidity values, using calculation procedures as given in design codes

Objective:	Comparison of estimated long term deflection values of a simply supported RC beam at different levels of relative humidity and loaded at early age of 6 days.
Design input:	of loading is limited to 6 days total. RH ranges from 20% - 100% and in 10% increments

Max Humidity (%)	**100**	**Step size (%)**	**10**	**Cement Type**	CEM42.5N
Age (days):	**3**	**Concrete Strength (Mpa)**	**30**		

Humidity (%)	**Long term deflection**		
	SANS 10100	**Eurocode**	**ACI 318**
20	21.6	31.9	21.2
30	21.2	30.1	20.4
40	20.5	28.2	19.6
50	19.7	26.4	18.8
60	18.8	24.6	17.9
70	17.6	22.7	17.1
80	16.2	20.8	16.3
90	14.4	18.7	15.3
100	12.3	16.9	14.2

Long Term Deflection
Humidity



Case Study 3: Comparison of deflections for different concrete strengths, using calculation procedures as given in design codes

Objective:	Comparison of estimated long term deflection values of a simply supported RC beam made from different concrete strengths, ranging from 25MPa to 60MPa
Design input:	Identical beam geometry and material properties were used in all calculations. RH is limited to 60% and age of loading is limited to 7 days. Cement type 42.5N

Max Strength (MPa)	60	**Step size (Mpa)**	5	**Cement Type**	CEM42.5N
Age (days):	7	**RH (%):**	60		

	Long term deflection		
Concrete strength (Mpa)	**SANS 10100**	**Eurocode**	**ACI 318**
25	16.0	27.1	18.0
30	15.6	25.5	17.4
35	15.2	23.9	17.1
40	14.8	21.9	16.7
45	14.5	20.4	16.4
50	14.2	19.1	16.1
55	13.9	18.0	15.9
60	13.6	17.1	15.7

Long Term Deflection
Concrete Strength